KENDRIYA VIDYALAYA SANGATHAN

Raipur Region

STUDENT SUPPORT MATERIAL

session 2020-21

तत् त्वं पूषन् अपावृणु
केन्द्रीय विद्यालय संगठन

Class XII

Mathematics

KENDRIYA VIDYALAYA SANGTHAN

REGIONAL OFFICE, RAIPUR

An Autonomous Body under the Ministry of Education, Government of India

STUDENT SUPPORT MATERIAL

MATHEMATICS

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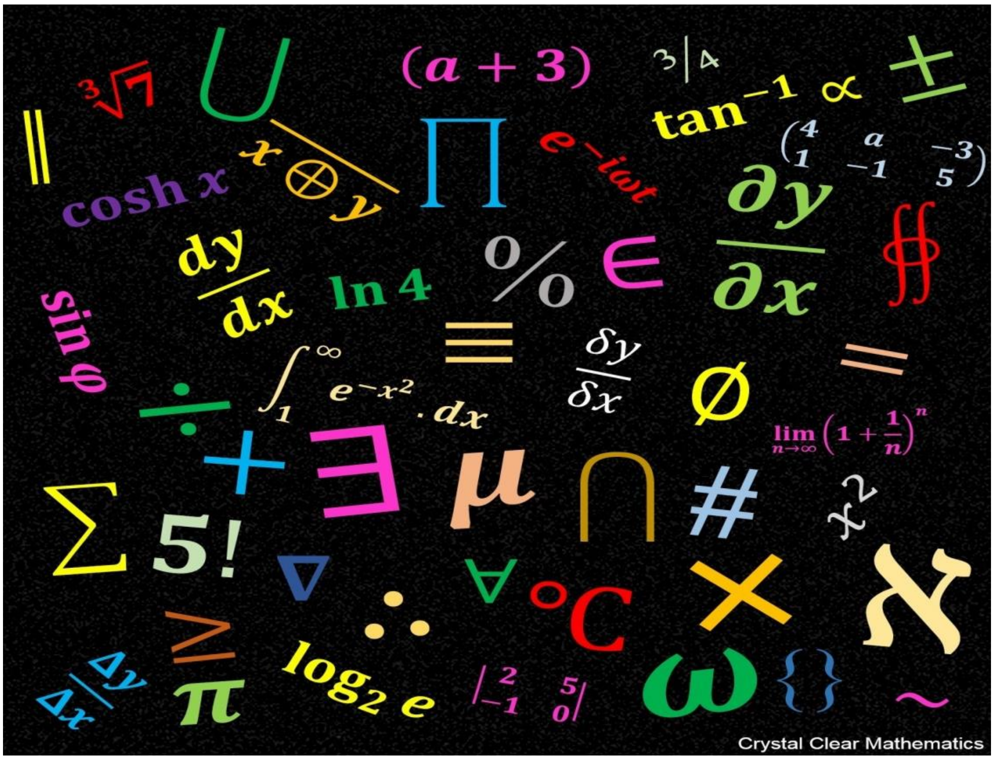

CLASS-XII

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RELATIONS AND FUNCTIONS

IMPORTANT POINT TO REMEMBER

1. Relation R from a set A to a set B is subset of A X B and Relation R I set A is a subset of A X A .
2. If n(A) = m , n(B)=n from set A to set B then n(A X B)= mn and number of relation = 2^{mn} .
3. $\emptyset$ is also a relation defined on set A , called the void (empty) relation .
4. R = A X A is called universal relation .
5. Reflexive Relation : Relation R defined on set A is side to be reflexive , if $(a , a) \in R \ \forall \ a \in A$.
6. Symmetric Relation : Relation R defined on set A is said to be symmetric if $(a , b) \in R \Rightarrow (b , a) \in R \ \forall \ a , b \in A$.
7. Transitive Relation : Relation R defined on set A is said to be transitive if $(a , b) \in R$, $(b , c) \in R \Rightarrow (a , c) \in R \ \forall \ a , b , c \in A$.
8. Equivalence Relation : A relation R on a set A is said to be an **equivalence relation** if and only if the relation R is reflexive, symmetric and transitive.
9. Equivalence Class of an element : Let R be an equivalence relation of set A, then equivalence class of $a \in A$ is $[a] =\{b \in A : (b , a) \in R \}$.
10. One – one Function : $f : A \rightarrow B$ is said to be one-one if distinct element in A have distinct images in B . i. e. $\forall x_1 , x_2 \in A$ such that $x_1 \neq x_2 \Rightarrow f(x_1) \neq f (x_2)$.

    OR

    $\forall x_1 , x_2 \in A$ such that $f(x_1) = f (x_2) \Rightarrow x_1 = x_2$

    One-one function is also called injective function .

11. Onto function (surjective) : A function $f : A \rightarrow B$ is said to be onto if $R_f = B$ i.e. $\forall b \in B$, there exists $a \in A$ such that $f(a) = b$.
12. Bijective Function : A function which is both injective and surjective is called bijective function .
13. Let A be any non-empty Set such that n(A) =n then
    a. Number of Relation on A = 2^n
    b. Number of Reflexive relation on A = $2^{n(n-1)}$.
    c. Number of Symmetric relations on A = $2^{n(n+1)/2}$.
    d. Number of Reflexive and Symmetric relations on A = $2^{n(n-1)/2}$
14. Lat A and B are two non-empty set that n(A) = p and n(B)=q then
    a. Number of functions from A to B = q^p
    b. Number of one-one functions from A to B = $\begin{cases} {}^qP_p & , p \leq q \\ 0 & , p > q \end{cases}$
    c. Number of onto function from A to B $\begin{cases} \sum_{r=1}^{q} (-1)^{q-r} \, {}^{q}C_r \, r^p , & p \geq q \\ 0 , & p < q. \end{cases}$
    d. Number of bijective functions from A to B = $\begin{cases} P!, & p=q \\ 0 , & p \neq q \end{cases}$

One Marks Question

1. If A is the set of students of a school then write, which of following relations are Universal, Empty or neither of the two.

   $R_1 = \{(a, b) : a, b$ are ages of students and $|a - b| > 0\}$

   $R_2 = \{(a, b) : a, b$ are weights of students, and $|a - b| < 0\}$

   $R_3 = \{(a, b) : a, b$ are students studying in same class$\}$

2. Is the relation R in the set $A = \{1, 2, 3, 4, 5\}$ defined as

   $R = \{(a, b) : b = a + 1\}$ reflexive?

3. If R, is a relation in set N given by

   $R = \{(a, b) : a = b - 3, b > 5\}$,

   then does element $(5, 7) \in R$?

4 If n(A) = n(B) = 3 , then how many bijective functions from A t o B can be formed ?

5 Is f: N → N given by $f(x) = x^2$, one - one ? Give Reason .

6. if f: R → A , given by $F(x) = x^2 - 2x + 2$ is onto function , find set A

7. If f:A → B is bijective function such that n(A) = 10 , then n(B) = ?

8. R = { (a , b) : a , b ∈ N , a ≠ b and a divides b } is R reflexive ? Give Reason .

9. is f: R → R , given by f(x) = |x – 1 | one – one ? Give reason .

10. f : R → B given by f (x) = sin x is onto function , then write set B .

11. If $f(x) = \log(1+x/1-x)$, show that $f(2x/(1+x^2)) = 2f(x)$

12 State the reason for the relation R in the set {1,2,3} given by R={(1,2),(2,1)} not to be transitive.

13. If R= {(x,y): x+2y = 8} is a relation on N, write the range of R.

14. Let A = { 0 , 1 , 2 , 3 } and a relation R on A as follows : R = { (0 ,0) (0 , 1) (0 , 3), (1 , 1) (2,2), (3 , 0) , (3 , 3) } is R reflexive ? Symmetric ? Transitive ?

15. Consider the Set A = { 1,2,3} . Write the Smallest equivalence relation R on A .

16. Let A= {1,2,3} and B= {4,5,6,7} and let f = {(1,4), (2,5), (3,6)} be a function from A to B. State whether f is one-one or not.

17. If A={1,2,3,4} and B={-1,3}, then what is the number of onto functions from A to B?

18 If A={-1,2,3} and B= {0,3,5} then what is the number of bijections from A to B?

19 If A= {-1,2,3} and B= {0,3,5,7} then what is the number of bijections from A to B?

TWO MARKS QUESTIONS

1. Check the following functions for one-one and onto : $f : R \to R,\ f(x) = \frac{2x-3}{7}$
2. Check the following functions for one-one and onto : $f : R \to R,\ f(x) = |x + 1|$
3. Check the following functions for one-one and onto : $f : R - \{2\} \to R,\ f(x) = \frac{3x-1}{x-2}$
4. Check the following functions for one-one and onto : $f : R - \{-1, 1\},\ f(x) = \sin^2 x.$
5. Let A = { 1 ,2 ,3} and define R = { (a , b) : a – b = 12 } . show that R is empty relation on set A .
6. Let A = { 1 ,2 ,3} and define R = { (a , b) : a + b > 0 } . show that R is inversal relation on set A .
7. Let A = { a, b ,c } how many relation can be define in the set ? How many of these are reflexive ?
8. Let f : R → R be defined by f(x) = x^2 + 1 , find the pre image of 17 and -3 .
9. Let A = { 2, 4 , 6 , 8} and R be the relation " is greater than " on the set A . Write R as a set of order pairs , is the relation reflexive ?
10. Let A = { 2, 4 , 6 , 8} and R be the relation " is greater than " on the set A . Write R as a set of order pairs , is the relation Symemtric ?
11. Let A = { 2, 4 , 6 , 8} and R be the relation " is greater than " on the set A . Write R as a set of order pairs , is the relation Transitive ?

Three Marks Questions

1. Let A = { 2, 4 , 6 , 8} and R be the relation " is greater than " on the set A . Write R as a set of order pairs , is the relation an equivalence Relation ?
2. Let T be the set of all triangles in a plane with R a relation in T given by R = {(T1, T2): T1 is congruent to T2}. Show that R is an equivalence relation.
3. Show that the relation R in the set Z of integers given by R={(a, b) : 2 divides a-b} is equivalence relation.
4. **Let L be the set of all lines in plane and R be the relation in L define if R = {(L1, L2): L1 is $\perp$ to L2 } . Show that R is symmetric but neither reflexive nor transitive.**
5. **Check whether the relation R defined in the set {1, 2, 3, 4, 5, 6} as R = {(a, b): b = a+1} is reflexive, symmetric or transitive.**
6. **Let f: N X N be defined by f(x) =** $\begin{Bmatrix} \frac{n+1}{2}, if\ n\ is\ odd\ for\ \text{all } n \in N \\ \frac{n}{2} if\ n\ is\ even \end{Bmatrix}$

Examine whether the function f is onto, one – one or bijective

7. Let A = R – {3} and B = R- {1}. Consider the function of f: A ⇨ B defined by f(x) = $\frac{x-2}{x-3}$. is f one – one and onto.

Five Marks Questions

1. **Show that the relation R defined by (a, b) R (c, d) $\Rightarrow$ a + b = b + c on the set N $\times$ N is an equivalence relation.**

2. Let N denote the set of all natural numbers and R be the relation on N N defined by (a, b) R (c, d) if ad (b + c) = bc (a + d). Show that R is
4. an equivalence relation.

3. Show that the relation R in the set A $= \{1, 2, 3, 4, 5\}$ given by R $= \{(a, b): |a - b| \text{ is divisible by } 2\}$ is an equivalence relation. Show that all the elements of $\{1, 3, 5\}$ are related to each other and all the elements of $\{2, 4\}$ are related to each other, but no element of $\{1, 3, 5\}$ is related to any element of $\{2, 4\}$.

4. Let A $= \{1,2,3, \dots, 9\}$ and R be the relation in A X A defined by (a, b) R (c , d) iff a + d = b +c for all a, b, c ,d $\in$ A .Prove that R is an equivalence relation. Also obtain the equivalence class [(2,5)]

5. Consider a function f: $R_+ \rightarrow [-5, \infty)$ given by f$(x) = 9x^2 + 6x - 5$, show that f is bijective function.

6. Consider a function f : $R_+ \rightarrow [4, \infty)$ is given by f $(x) = x^2 + 4$. Show that f is bijective function

7. Consider a function f : $R_+ \rightarrow [15, \infty)$ given by f $(x) = 4x^2 + 12x + 15$. Show that f is bijective function

CASE STUDY QUESTIONS

Q. 1. In two different societies, there are some school going students - including girls as well as boys.

Satish forms two sets with these students, as his college project.

Let A a_1, a_2, a_3, a_4, a_5 and B b_1, b_2, b_3, b_4 where a_i 's and b_i 's are the school going students of first and second society respectively.

Satish decides to explore these sets for various types of relations and functions.

Using the information given above, answer the following :

A . Satish wishes to know the number of reflexive relations defined on set A. How many such relations are possible?

1. 0
2. 2^5
3. 2^{10}
4. 2^{20}

B. Let R : A→ A, R { (x, y) : x and y are students of same sex }. Then relation R is

1. reflexive only
2. reflexive and symmetric but not transitive
3. reflexive and transitive but not symmetric
4. an equivalence relation

C. Satish and his friend Rajat are interested to know the number of symmetric relations defined on both the sets A and B, separately. Satish decides to find the symmetric relation on set A, while Rajat decides to find the symmetric relation on set B. What is difference between their results?

1. 1024
2. 2^{10} (15)
3. 2^{10} (31)
4. 2^{10} (63)

D. Let R : A → B , R = { (a_1, b_1), (a_2, b_1), (a_3, b_3), (a_4, b_2), (a_5, b_2)} , then R is

1. neither one-one nor onto
2. one-one but, not onto
3. only onto, but not one-one
4. one-one and onto both

E. To help Satish in his project, Rajat decides to form onto function from set A to B. How many such functions are possible?

1. 342
2. 240
3. 729
4. 1024

Q.2 Amit and Vivek are students of class XII . Their maths teacher told them to collect the names of 5 students of class X and 4 students of class IX , Amit collected the names of students is denoted by A = { Anshul , Garima , Aditi , Shravan, Nitin } and Vivek collected the names of students denoted by B = { Rajat , Jagriti ,Ankush , Avi } . Since discussion of Relation and function was giving as the class . From the above information give the answer of following question .

1. How many functions exist from A to B
   a. 20
   b. 2^{20}
   c. 1024
   d. 625
2. If you want to know no. of relations exist from A to B . How many such relations possible ?
   a. 20
   b. 2^{20}
   c. 5^4
   d. 4^5
3. Let R: A→ A defined by R={ (x , y) : total marks obtained by x is less then the total marks obtained by y the R is
   a. Reflexive and Symmetric
   b. Symmetric and Transitive
   c. Equivalence Relation
   d. None of these
4. How many Reflexive and Symmetric relations exist on Set A
   a. 2^{10} b. 2^{15} c. 2^{20} d. 2^5

5. How many Symmetric relations exist on Set B
   b. 2^{15} b. 2^{10} c. 2^{20} d. 2^5

Answer Key

One Marks Question :

1. R_1 : is universal relation.

R_2 : is empty relation.

R_3 : is neither universal nor empty.

2. No, R is not reflexive.

3. $(5, 7) \notin R$

4. 6

5. Yes, f is one-one $\because \forall x_1, x_2 \in N \Rightarrow {x_1}^2 = {x_2}^2$.

6. $A = [1, \infty)$ because $R_f = [1, \infty)$

7. $n(B) = 10$

8. R is reflexive as a divides a.

9. Not one one because -1 and 3 have the same image.

10. $B = [-1, 1]$

12. (1,2) ∈ R and (2, 1) ∈ R but (1, 1) ∈ R

13. Range = { 1 ,2 , 3 }

14. **Reflexive and Symmetric but Not Transitive**
15. **{ (1, 1), (2, 2) , (3 ,3)}**
16. **Yes**
17. **14**
18. **6**
19. **0**

Two Marks Question

1. **Bijective (one-one , onto)**
2. **Neither one-one nor onto**
3. **One-one but not onto**
4. **Neither one-one nor onto**

7. **512 , 64**
8. ± 4, pre image of -3 does not exist.
9. R = {(8, 6), (8, 4), (8, 2) (6, 4), (6, 2), (4, 2)} Not reflexive
10. R = {(8, 6), (8, 4), (8, 2) (6, 4), (6, 2), (4, 2) Not symmetric
11. R = {(8, 6), (8, 4), (8, 2) (6, 4), (6, 2), (4, 2) Not Transitive

Three Marks Question

1. **Not Equivalence Relation**

5. **Neither Reflexive nor Symmetric and Not Transitive .**
6. **F is onto ; f is not one-one , f is not bijective .**
7. **F is one-one and onto**

Five marks Question

4. **[(1 , 4) (2 , 5) (3 , 6) (4 , 7) (5 , 8) (6 , 9)]**

Case Study Question

1. **(A.) 4 (B.) 4 (C.) 3 (D.) 1 (E.) 2**
2. **(1) C (2) B (3) D (4) A**
   (5) B

INVERSE TRIGONOMETRIC FUNCTIONS

IMPORTANT POINTS TO REMEMBER

Functions	**Domain**	**Range**
Sin^{-1} x	[-1, 1]	[-π/2, π/2]
Cos^{-1}x	[-1, 1]	[0, π/2]
Tan^{-1} x	R	(-π/2, π/2)
Cosec^{-1} x	R-(-1,1)	[-π/2, π/2]
Sec^{-1} x	R-(-1,1)	[0,π]-{ π/2}
Cot^{-1} x	R	[-π/2, π/2]-{0}

S.No	**Inverse Trigonometric Formulas**
1	$\sin^{-1}(-x) = -\sin^{-1}(x), x \in [-1, 1]$
2	$\cos^{-1}(-x) = \pi - \cos^{-1}(x), x \in [-1, 1]$
3	$\tan^{-1}(-x) = -\tan^{-1}(x), x \in R$
4	$\text{cosec}^{-1}(-x) = -\text{cosec}^{-1}(x), \lvert x\rvert \geq 1$
5	$\sec^{-1}(-x) = \pi - \sec^{-1}(x), \lvert x\rvert \geq 1$
6	$\cot^{-1}(-x) = \pi - \cot^{-1}(x), x \in R$
7	$\sin^{-1}x + \cos^{-1}x = \pi/2 , x \in [-1, 1]$
8	$\tan^{-1}x + \cot^{-1}x = \pi/2 , x \in R$
9	$\sec^{-1}x + \text{cosec}^{-1}x = \pi/2 , \lvert x\rvert \geq 1$
10	$\sin^{-1}(1/x) = \text{cosec}^{-1}(x)$, if $x \geq 1$ or $x \leq -1$
11	$\cos^{-1}(1/x) = \sec^{-1}(x)$, if $x \geq 1$ or $x \leq -1$
12	$\tan^{-1}(1/x) = \cot^{-1}(x), x > 0$
13	$\sin(\sin^{-1}(x)) = x, -1 \leq x \leq 1$

14	$\cos(\cos^{-1}(x)) = x, -1 \le x \le 1$
15	$\tan(\tan^{-1}(x)) = x, -\infty < x < \infty$.
16	$\operatorname{cosec}(\operatorname{cosec}^{-1}(x)) = x, -\infty < x \le 1$ or $-1 \le x < \infty$
17	$\sec(\sec^{-1}(x)) = x, -\infty < x \le 1$ or $1 \le x < \infty$
18	$\cot(\cot^{-1}(x)) = x, -\infty < x < \infty$.
19	$\sin^{-1}(\sin \theta) = \theta, -\pi/2 \le \theta \le \pi/2$
20	$\cos^{-1}(\cos \theta) = \theta, 0 \le \theta \le \pi$
21	$\tan^{-1}(\tan \theta) = \theta, -\pi/2 < \theta < \pi/2$
22	$\operatorname{cosec}^{-1}(\operatorname{cosec} \theta) = \theta, -\pi/2 \le \theta < 0$ or $0 < \theta \le \pi/2$
23	$\sec^{-1}(\sec \theta) = \theta, 0 \le \theta \le \pi/2$ or $\pi/2 < \theta \le \pi$
24	$\cot^{-1}(\cot \theta) = \theta, 0 < \theta < \pi$

One Mark Question

1. Write the Principal Value of : $\sin^{-1}(-\sqrt{3}/2)$ Ans : $-\pi/3$
2. Write the Principal Value of : $\cos^{-1}(-\sqrt{3}/2)$ Ans : $5\pi/6$
3. Write the Principal Value of : $\tan^{-1}(-1/\sqrt{3})$ Ans : $-\pi/6$
4. Write the Principal Value of : $\operatorname{cosec}^{-1}(-2)$ Ans : $-\pi/6$
5. Write the Principal Value of : $\cot^{-1}(1/\sqrt{3})$ Ans : $\pi/3$
6. Write the Principal Value of : $\sec^{-1}(-2)$ Ans : $2\pi/3$
7. Find the principal value of : $\sin^{-1}\left(\sin\frac{4\pi}{5}\right)$ Ans : $\pi/5$
8. Find the principal value of : $\tan^{-1}\left(\tan\frac{5\pi}{6}\right)$ Ans : $-\pi/6$
9. Find the principal value of : $\operatorname{cosec}^{-1}\left(\operatorname{cosec}\frac{3\pi}{4}\right)$ Ans : $\pi/4$
10. Find the Principal Value of : $\tan^{-1}(\sin(-\pi/2))$ Ans : $-\pi/4$

Two Marks Question

1. If $\tan^{-1} x + \tan^{-1} y = 4\pi/5$ find $\cot^{-1}x + \cot^{-1} y$ Ans : $\pi/5$

2 Find the Principal value of : $\sin\{\pi/6 - \sin^{-1}(-\sqrt{3}/2)\}$ Ans : 1

3. Find the Principal Value of : $\sin^{-1}(\cos(\sin^{-1}\sqrt{3}/2))$ Ans : $\pi/6$

4. Write the principal value of :- $4\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right)$

Ans: $\frac{5\pi}{3}$

5. Find the Principal value of : $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \cos^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ Ans : $\pi/6$

6. Find the Principal value of : $\sin^{-1}\left(\sin\frac{2\pi}{3}\right) + \cos^{-1}\left(\cos\frac{4\pi}{3}\right)$ Ans : π

7. Find the Principal value of : $\sin\left\{\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right\}$ Ans : 1

8. Find the Principal value of : $\tan^{-1}\left(\tan\frac{7\pi}{6}\right) - \cos^{-1}\left(\cos\frac{7\pi}{6}\right)$ Ans : $-\pi$

9. Simplify : $\tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$ Ans : $x/2$

10. Simplify : $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right), x < -1$ Ans : $\pi - \sec^{-1}x$

Three Marks Question

1. Simplify : $\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$ **Ans:** $\sin^{-1}\left(\frac{x}{a}\right)$

2. Simplify : $\tan^{-1}\left[2\cos\left\{2\sin^{-1}\left(\frac{1}{2}\right)\right\}\right]$ **Ans:** $\frac{\pi}{4}$

3. Write $\sin^{-1}(2x\sqrt{1-x^2})$ in the simplest form. **Ans:** $2\sin^{-1}x$

4. Show that : $\tan^{-1}\left[\frac{\sqrt{1+\cos x} + \sqrt{1-\cos x}}{\sqrt{1+\cos x} - \sqrt{1-\cos x}}\right] = \frac{\pi}{4} + \frac{x}{2};\ x \in [0, \pi]$

5. Prove that $\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right) = \sin^{-1}\frac{x}{a} = \cos^{-1}\left(\frac{\sqrt{a^2-x^2}}{a}\right)$.

MATRICES

Types of Matrices

(i) A matrix is said to be a row matrix if it has only one row.

(ii) A matrix is said to be a column matrix if it has only one column.

(iii) A matrix in which the number of rows are equal to the number of columns,is said to be a square matrix. Thus, an $m \times n$ matrix is said to be a squarematrix if $m = n$ and is known as a square matrix of order 'n'.

(iv) A square matrix $B = [b_{ij}]_{n\times n}$ is said to be a diagonal matrix if its all nondiagonal elements are zero, that is a matrix $B = [b_{ij}]_{n\times n}$ is said to be adiagonal matrix if $b_{ij} = 0$, when $i \neq j$.

(v) A diagonal matrix is said to be a scalar matrix if its diagonal elements areequal, that is, a square matrix $B = [b_{ij}]_{n\times n}$ is said to be a scalar matrix if$b_{ij}= 0$, when $i \neq j$$b_{ij} = k$, when $i = j$, for some constant k.

(vi) A square matrix in which elements in the diagonal are all 1 and rest are

all zeroes is called an identity matrix.$a_{ij}= 1$, when $i = j$ and $a_{ij} = 0$, when $i \neq j$.

(vii) A matrix is said to be zero matrix or null matrix if all its elements arezeroes. We denote zero matrix by O.

(ix) Two matrices $A = [a_{ij}]$ and $B = [b_{ij}]$ are said to be equal if(a) they are of the same order, and

(b) each element of A is equal to the corresponding element of B, that is,$a_{ij} = b_{ij}$ for all i and j.

Transpose of a Matrix

1. If $A = [a_{ij}]$ be an $m \times n$ matrix, then the matrix obtained by interchangingthe rows and columns of A is called the transpose of A.Transpose of the matrix A is denoted by A′ or (A^T). In other words, if$A = [a_{ij}]_{m\times n}$, then $A^T= [a_{ji}]_{n\times m}$.

2. Properties of transpose of the matrices

(i) $(A^T)^T= A$,(ii) $(kA)^T = kA^T$ (where k is any constant)(iii) $(A + B)^T = A^T + B^T$(iv) $(AB)^T = B^TA^T$

Symmetric Matrix and Skew Symmetric Matrix

(i) A square matrix A = $[a_{ij}]$ is said to be symmetric if $A^T = A$, that is,$a_{ij} = a_{ji}$for all possible values of i and

(ii) A square matrix A = $[a_{ij}]$ is said to be skew symmetric matrix if $A^T= –A$,that is $a_{ji} = –a_{ij}$ for all possible values of i and j.

Theorem 1: For any square matrix A with real number entries, $A + A^T$isa symmetric matrix and $A – A^T$ is a skew symmetric matrix.

Theorem 2: Any square matrix A can be expressed as the sum of asymmetric matrix and a skew symmetric matrix, that is $(A + A^T)/2 +(A – A^T)/2$

Theorem 3 (Uniqueness of inverse) Inverse of a square matrix, if itexists, is unique.

Theorem 4 : If A and B are invertible matrices of same order, then$(AB)^{–1} = B^{–1}A^{–1}$.

Invertible Matrices

(i) If A is a square matrix of order m × m, and if there exists another squarematrix B of the same order m × m, such that AB = BA = I_m, then, A is saidto be invertible matrix and B is called the inverse matrix of A and it isdenoted by $A^{–1}$.

Very Short Answer Question

Q1 If A and B are square matrices of the same order, then(A + B) (A – B) is equal to…………

Ans: $A^2– AB + BA – B^2$

Q2If A and B are two skew symmetric matrices of same order, then AB is symmetric matrix if _.

Ans : AB = BA.

Q3If a matrix has 28 elements, what are the possible orders it can have? What if it has 13 elements?

Q4If A is square matrix such that $A^2= A$, show that $(I + A)^3= 7A + I$.

Q5 Total number of possible matrices of order 3 × 3 with each entry 2 or 0 is…….

Ans 512

Q6If A and B are two matrices of the order 3 × m and 3 × n, respectively, and m = n, then the order of matrix (5A – 2B) is

Ans

Q7 If $A=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$ is identity matrix. Then write the value of α

Ans : α = 0

Q8 If $\begin{bmatrix}2x+y & 3y\\ 0 & 4\end{bmatrix}=\begin{bmatrix}6 & 0\\ 6 & 4\end{bmatrix}$, then find x

Ans : x=3 , y = 0

Q9 If $A=\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$ satisfies $A^4=\lambda A$ then write the value of λ

Ans : λ = 0

Q10 Construct a 2 x 2 matrix $A=\lfloor a_{ij}\rfloor$ whose elements are given by $a_{ij}=\frac{(i+2j)^2}{2}$.

Ans : $\begin{bmatrix}\frac{9}{2} & \frac{25}{2}\\ 8 & 18\end{bmatrix}$

Case Study type questions

Q1. Suppose Meera and Nadeem are two friends. Meera wants to buy 2 pens and 5 story books, while Nadeem needs 8 pens and 10 story books. They both go to a first shop to enquire about the rates which are quoted as follows: Pen – ` 5 each, story book – ` 50 each. Suppose that they enquire about the rates from second shop, quoted as follows: pen – ` 4 each, story book – ` 40 each.

Q1How much money does Meera need to spend in first shop ?

(A) Rs 270 (B) Rs 260 (c) Rs 280 (D) Rs 540

Q2How much money does Nadeem need to spend in second shop?

(A) Rs 208 (B) Rs 540 (c) Rs 432 (D) Rs 260

Q3In term of matrix How much money does Meera and Nadeem need to spend in first shop ?

(A) $\begin{bmatrix} 260 \\ 540 \end{bmatrix}$ (B) $\begin{bmatrix} 432 \\ 260 \end{bmatrix}$ (c) $\begin{bmatrix} 260 \\ 208 \end{bmatrix}$ (D) $\begin{bmatrix} 540 \\ 432 \end{bmatrix}$

Q4the information in both the cases can be combined and expressed in terms of matrices as follows:(A) $\begin{bmatrix} 2 & 5 \\ 8 & 10 \end{bmatrix}\begin{bmatrix} 5 & 50 \\ 4 & 40 \end{bmatrix} = \begin{bmatrix} 260 & 540 \\ 208 & 432 \end{bmatrix}$ (B) $\begin{bmatrix} 2 & 5 \\ 8 & 10 \end{bmatrix}\begin{bmatrix} 5 & 50 \\ 4 & 40 \end{bmatrix} = \begin{bmatrix} 540 & 260 \\ 208 & 432 \end{bmatrix}$

(C) $\begin{bmatrix} 2 & 5 \\ 8 & 10 \end{bmatrix}\begin{bmatrix} 5 & 50 \\ 4 & 40 \end{bmatrix} = \begin{bmatrix} 432 & 540 \\ 208 & 260 \end{bmatrix}$ (D) $\begin{bmatrix} 2 & 5 \\ 8 & 10 \end{bmatrix}\begin{bmatrix} 5 & 50 \\ 4 & 40 \end{bmatrix} = \begin{bmatrix} 260 & 208 \\ 540 & 432 \end{bmatrix}$

Ans (1)B (2) C (3) A (4) A

SHORT ANSWER QUESTIONS

Q1. If $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then find k so that $A^2 = 8A + kI$

Ans : K = -7

Q2. Find the values of x and y from the following equation:

$$2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$$

Ans: $x = 2, y = 9$

Q3. If A and B are symmetric matrices, prove that AB – BA is a skew symmetric matrix.

Q4Find the matrix X so that $X\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

Ans : $X = \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$

Q5 Find $\frac{1}{2}(A+A')$ and $\frac{1}{2}(A-A')$, When $A=\begin{bmatrix}0 & a & b\\ -a & 0 & c\\ -b & -c & 0\end{bmatrix}$

Ans : $\frac{1}{2}(A+A')=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$ and $\frac{1}{2}(A-A')=\begin{bmatrix}0 & a & b\\ -a & 0 & c\\ -b & -c & 0\end{bmatrix}$

Q6 If $A=\begin{bmatrix}-2\\ 4\\ 5\end{bmatrix}, B=[1 \quad 3 \quad -6]$,verify that $(AB)' = B'A'$

Q7If $[2x \quad 3]\begin{bmatrix}1 & 2\\ -3 & 0\end{bmatrix}\begin{bmatrix}x\\ 8\end{bmatrix}=0$, find the value of x. Ans $x=0, x=\frac{-23}{2}$

Q8If A = [3 5], B = [7 3], then find a non-zero matrix C such that AC = BC.Ans :

Q9 If A is 3 × 3 invertible matrix, then show that for any scalar k (non-zero),kA is invertible and $(KA)^{-1}=\frac{1}{k}A^{-1}$.

Q10Construct a matrix A = $[a_{ij}]_{2\times 2}$ whose elements $a_{ij}$are given bya_{ij} = $e^{2ix}\sin jx$.

Ans $\begin{bmatrix}e^{2x}\sin x & e^{2x}\sin 2x\\ e^{4x}\sin x & e^{4x}\sin 2x\end{bmatrix}$

LONG ANSWER QUESTIONS

Q1Express the matrix A as the sum of a symmetric and a skew symmetric matrix, where

$$A=\begin{bmatrix}2 & 4 & -6\\ 7 & 3 & 5\\ 1 & -2 & 4\end{bmatrix}$$

Q2Let $A=\begin{bmatrix}2 & 3\\ -1 & 2\end{bmatrix}$Then show that A^2– 4A + 7I = O.Using this result calculate A^5 also.

Ans : $A^5=\begin{bmatrix}-118 & -93\\ 31 & -118\end{bmatrix}$

Q3 If AB = BA for any two square matrices, prove by mathematical induction that $(AB)^n = A^n B^n$.

Q4 Let $A=\begin{bmatrix}2 & -1\\ 3 & 4\end{bmatrix}, B=\begin{bmatrix}5 & 2\\ 7 & 4\end{bmatrix}, C=\begin{bmatrix}2 & 5\\ 3 & 8\end{bmatrix}$. Find a matrix D such that CD - AB = O .

Ans $\begin{bmatrix}-191 & -110\\ 77 & 44\end{bmatrix}$

Q5Find the value of x such that $\begin{bmatrix}1 & x & 1\end{bmatrix}\begin{bmatrix}1 & 3 & 2\\ 2 & 5 & 1\\ 15 & 3 & 2\end{bmatrix}\begin{bmatrix}1\\ 2\\ x\end{bmatrix}=0$ Ans : x= -2 or -14

Q6 Prove the following by the principle of mathematical induction: If $A=\begin{bmatrix}3 & -4\\ 1 & -1\end{bmatrix}$, then $A^n=\begin{bmatrix}1+2n & -4n\\ n & 1-2n\end{bmatrix}$ for every positive integer n.

DETERMINANTS

Area of a triangle

Area of a triangle with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Minors and co-factors

(i) Minor of an element a_{ij} of the determinant of matrix A is the determinant obtained

by deleting i^{th} row and j^{th} column, and it is denoted by M_{ij}.

(ii) Co-factor of an element a_{ij} is given by $A_{ij}= (-1)^{i+j}M_{ij}$.

(iii) Value of determinant of a matrix A is obtained by the sum of products of elementsof a row (or a column) with corresponding co-factors. $|A| = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$.

(iv) If elements of a row (or column) are multiplied with co-factors of elements of

any other row (or column), then their sum is zero.,$a_{11} A_{21} + a_{12} A_{22} + a_{13} A_{23} = 0$.

Adjoint and inverse of a matrix

(i) The adjoint of a square matrix $A = [a_{ij}]_{n\times n}$is defined as the transpose of the matrix

$[a_{ij}]_{n\times n}$, where $A_{ij}$is the co-factor of the element a_{ij}. It is denoted by adj A.

(ii) A (adj A) = (adj A) A = $|A|$ I, where A is square matrix of order n.

(iii) A square matrix A is said to be singular or non-singular according as $|A| = 0$ or$|A| \neq 0$, respectively.

(iv) If A is a square matrix of order n, then $|\text{adj } A| = |A|^{n-1}$.

(v) If A and B are non-singular matrices of the same order, then AB and BA arealso nonsingular matrices of the same order.

(vi) The determinant of the product of matrices is equal to product of their respective

determinants, that is, |AB| = |A| |B|.

(vii) If AB = BA = I, where A and B are square matrices, then B is called inverse ofA and is written as $B = A^{-1}$. Also $B^{-1} = (A^{-1})^{-1} = A$.

(viii) A square matrix A is invertible if and only if A is non-singular matrix.

(ix) If A is an invertible matrix, then $A^{-1} = \frac{adjA}{|A|}$

System of linear equations

(i) Consider the equations: $a_1x + b_1 y + c_1 z = d_1$,$a_2x + b_2 y + c_2 z = d_2$,$a_3x + b_3 y + c_3 z = d_3$

,In matrix form, these equations can be written as A X = B, whereUnique solution of equation AX = B is given by $X = A^{-1}B$, where $|A| \neq 0$.For a square matrix A in matrix equation AX = B

(a) If $|A| \neq 0$, then there exists unique solution.

(b) If |A| = 0 and (adj A) B ≠ 0, then there exists no solution.

(c) If |A| = 0 and (adj A) B = 0, then system may or may not be consistent.

VERY SHORT ANSWER QUESTION

Q1Find values of x for which $\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$. Ans : $x = 2\sqrt{2}$

Q2Find the area of the triangle whose vertices are (3, 8), (– 4, 2) and (5, 1).

Ans : 61/2.

Q3Find minors and cofactors of all the elements of the determinant $\begin{vmatrix} 1 & -2 \\ 4 & 3 \end{vmatrix}$.

Q4 For what value of x is the matrix $\begin{bmatrix} 6-x & 4 \\ 3-x & 1 \end{bmatrix}$ singular? Ans x = 2

Q5 A matrix A of order 3x3 is such that |A| = 4 .Find the value of |2A|. Ans 32

Q6 Evaluate $\begin{vmatrix} \cos 15^0 & \sin 15^0 \\ \sin 75^0 & \cos 75^0 \end{vmatrix}$ Ans : 0

Q7.If A is 3x3 matrix, $|A| \neq 0$ and $|3A| = K|A|$ then write the value of k Ans : 27

Q8 Write the cofactor of the element a_{32} . If $A = \begin{bmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{bmatrix}$ Ans : 11

Q9 If the points (2,-3),(λ,-1) and (0,4) are collinear, find the value of λ Ans : λ= 10/7

Q10.If $A = \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$,find the determinant of the matrix $A^2 - 2A$ Ans : 25

SHORT ANSWER QUESTIONS

Q1If $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$,then verify that $A.adjA = |A|I$,also find A^{-1} Ans:

$$A^{-1} = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}.$$

Q2 If $A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 7 \\ 8 & 9 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1}A^{-1}$.

Q3Show that $A = \begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix}$ satisfies the equation $x^2 - 6x + 17 = 0$. Hence find A^{-1}.

Ans: $\frac{1}{17}\begin{bmatrix} 4 & 3 \\ -3 & 2 \end{bmatrix}$

Q4Find the non -singular matrices A,if its is given that $adjA = \begin{bmatrix} -1 & -2 & 1 \\ 3 & 0 & -3 \\ 1 & -4 & 1 \end{bmatrix}$.

Ans: $A = \pm\frac{1}{\sqrt{3}}\begin{bmatrix} -6 & -1 & 3 \\ -3 & -1 & 0 \\ -6 & -3 & -3 \end{bmatrix}$

Q5For the matrix $A=\begin{bmatrix}3 & 1\\ 7 & 5\end{bmatrix}$, find x and y so that $A^2+xI=yA$. also find A^{-1}.

Ans : x=8,y=8 $A^{-1}=\frac{1}{8}\begin{bmatrix}5 & -1\\ -7 & 3\end{bmatrix}$

Q6 Find the inverse of the matrix $A=\begin{bmatrix}2 & 1 & 3\\ 4 & -1 & 0\\ -7 & 2 & 1\end{bmatrix}$ Ans : $A^{-1}=\frac{-1}{3}\begin{bmatrix}-1 & 5 & 3\\ -4 & 23 & 12\\ 1 & -11 & -6\end{bmatrix}$

Q7.For the matrix $A=\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & -3\\ 2 & -1 & 3\end{bmatrix}$ show that $A^3-6A^2+5A+11I=O$ hence , find A^{-1}.

Ans: $A^{-1}=\frac{1}{11}\begin{bmatrix}-3 & 4 & 5\\ 9 & -1 & -4\\ 5 & -3 & -1\end{bmatrix}$

Q8 If $A=\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$, find $(adjA)^{-1}, \& (adjA^{-1})$ Ans: $A=\frac{1}{4}\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$

Q9If $A=\begin{bmatrix}1 & \tan x\\ -\tan x & 1\end{bmatrix}$,show that $A^T A^{-1}=\begin{bmatrix}\cos 2x & -\sin 2x\\ \sin 2x & \cos 2x\end{bmatrix}$

Q10.If $A=\begin{bmatrix}1 & -1 & 1\\ 2 & -1 & 0\\ 1 & 0 & 0\end{bmatrix}$, Show that $A^2=A^{-1}$.

Long Answer questions

Q1 Determine the product $\begin{bmatrix}-4 & 4 & 4\\ -7 & 1 & 3\\ 5 & -3 & -1\end{bmatrix}\begin{bmatrix}1 & -1 & 1\\ 1 & -2 & -2\\ 2 & 1 & 3\end{bmatrix}$ and use it to solve the system of equations $x-y+z=4, x-2y-2z=9, 2x+y+3z=1$ Ans x = 3 , y = -2 & z = -1

Q2. Given $A=\begin{bmatrix}2 & 2 & -4\\ -4 & 2 & -4\\ 2 & -1 & 5\end{bmatrix}, B=\begin{bmatrix}1 & -1 & 0\\ 2 & 3 & 4\\ 0 & 1 & 2\end{bmatrix}$,Find BA and use of this to solve the system of equations: $y+2z=7, x-y=3, 2x+3y+4z=17$ Ans: x = 2 ,y = -1 , z = 4.

Q3 Use the product $\begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{bmatrix}\begin{Vmatrix}-2 & 0 & 1\\ 9 & 2 & -3\\ 6 & 1 & -2\end{Vmatrix}$ to solve the system of equations

$x+3z=-9, -x+2y-2z=4, 2x-3y+4z=-3$

Q4Solve the following system of equations by matrix method.

3x – 2y + 3z = 8, 2x + y – z = 1, 4x – 3y + 2z = 4

Ans :x = 1, y = 2 and z = 3.

Q5 The sum of three numbers is 6. If we multiply third number by 3 and addsecond number to it, we get 11. By adding first and third numbers, we get double of thesecond number. Represent it algebraically and find the numbers using matrix method.

Ans :x = 1, y = 2, z = 3

Q6 If $A=\begin{bmatrix}2 & -3 & 5\\ 3 & 2 & -4\\ 1 & 1 & -2\end{bmatrix}$ find A^{-1}. Using A^{-1} solve the system of equations

2x – 3y + 5z = 11 ,3x + 2y – 4z = – 5 ,x + y – 2z = –3. Ans : x = 1, y = 2 , z = 3

Q7Solve the following system of equations by matrix method.

$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4, \frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1, \frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2$ Ans x = 2, y = 3 , z = 5

Q8Solve the following system of equations by matrix method

3x + 2y – 2z = 3, x + 2y + 3z = 6, 2x – y + z = 2Ans : x =1 , y = 1 ,z = 1

Continuity and Differentiability

LEARNING OBJECTIVES/OUTCOMES

Understanding the concept of Continuity and differentiability and addressing the problems based on continuity
and derivative of composite functions, chain rule, derivatives of inverse trigonometric functions, derivative of
implicit functions.

Learning the concept of exponential and logarithmic functions.

Skills to solve derivatives of logarithmic and exponential function. Logarithmic differentiation, derivative of
functions expressed in parametric forms. Second order derivatives, Rolle‘s and Lagrange‘s Mean Vale
Theorems and their geometric interpretation.

1. **Knowledge of functions :**
   (i) Polynomial functions: e.g. f(x) = $x^2 + 2x + 5$
   (ii) Modulus function : f(x) = $|x|$
   (iii) Greatest Integer Function : f(x) = [x]
   (iv) Signum function : The signum function, denoted sgn, is defined as follows:
   $$sgn(x) = \begin{cases} 1, & x > 0 \\ -1, & x < 0 \\ 0, & x = 0 \end{cases}$$
   (v) Trigonometric functions : sin x, cos x etc.
   (vi) Inverse Trigonometric functions : $\sin^{-1} x$, $\cos^{-1} x$ etc.
   (vii) Logarithmic functions : f(x) = log x
   (viii) Exponential functions : f(x) = e^x

Limits

1. $\lim_{\theta \to 0} \cos\theta = 1$
2. $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$
3. $\lim_{\theta \to 0} \frac{\tan\theta}{\theta} = 1$
4. $\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$
5. $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$
6. $\lim_{x \to a} \frac{\log(1+x)}{x} = 1$
7. Algebra of Limits: $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$
   (i) If $\lim_{x \to a} g(x)$=0 and $\lim_{x \to a} f(x) \neq 0$ $then$ limit does not exist
   (ii) If $\lim_{x \to a} f(x)$=0 and $\lim_{x \to a} g(x) \neq 0$ $then$ limit of the function is zero
   (iii) If $\lim_{x \to a} f(x)$=0 and $\lim_{x \to a} g(x)$=0 then f(x) , g(x) can be factorized or rationalized or simplified using trigonometric identities

Continuity

Definition -1

A real function f(x) on a subset of real numbers and let a be a point in the domain of f then f is continuous at a if $\lim_{x\to a} f(x) = f(a)$

Example: Find if $\lim_{x\to 0} (x-5)$

Deinition-2

A real function f(x) is said to be continuous at a if $\lim_{x\to a-} f(x) = \lim_{x\to a+} f(x) = f(a)$

Example: Verify that the following function f(x) is continuous at x = 2

$$f(x) = \begin{cases} 2x+3, & x \le 2 \\ 2x-3, & x > 2 \end{cases}$$

Theorem

Suppose f and g are real valued function such that fog is defined at c. If g is continuous at c and if f is continuous at g(c) then fog is continuous at c

Note

If the point a is not in the domain of f, we do not talk about whether or not f is continuous at a.

Continuous on a Subset of the Domain

The function f is **continuous on the subset S of its domain** if it continuous at each point of S.

Note:

1. All polynomial functions are continuous.
2. All rational functions are continuous provided denominator does not vanish.
3. All trigonometric functions are continuous.
4. All exponential functions are continuous.

Differentiability

1) The **derivative** of the function f at the point a in its domain is given by $f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$

2) The function f is **differentiable at the point a in its domain** if f'(a) exists.

3) The function f is **differentiable on the subset S of its domain** if it differentiable at each point of S.

4) A function can fail to be differentiable at a point a if either $\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$ does not exist, or is infinite

Note

(a) Not all continuous functions are differentiable. For instance, the closed-form function f(x) = |x| is continuous at every real number (including x = 0), but not differentiable at x = 0. (b) However, every differentiable function is continuous.

Derivatives of different types of functions :

Y=f(x)	$\frac{dy}{dx} = f'(x)$
C (constant)	0
x^n	nx^{n-1}
sinx	Cosx
cosx	-sinx
tanx	Sec^2x
secx	secxtanx
cosecx	-cosecxcotx
cotx	$-cosec^2x$
e^x	e^x
logx	1/x
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$\frac{-1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\frac{1}{1+x^2}$

$\cot^{-1} x$	$\frac{-1}{1+x^2}$
$\sec^{-1} x$	$\frac{1}{x\sqrt{x^2-1}}$
$\text{cosec}^{-1} x$	$\frac{-1}{x\sqrt{x^2-1}}$
(u+v)	u'+v'
u-v	u'-v'
uv	uv'+vu'
$\frac{u}{v}$	$\frac{vu'-uv'}{v^2}$
$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$
$\frac{1}{x}$	$\frac{-1}{x^2}$
a^x	a^xloga

Chain Rule

Let y=vou, t = u(x) and if both $\frac{dt}{dx}$, $\frac{dv}{dt}$ exist then $\frac{dy}{dx}=\frac{dv}{dt}.\frac{dt}{dx}$

Derivatives of Implicit and explicit functions

Example: $y = sinx$ + 2x (Explicit function)

$x + sinxy - y = 0$ (implicit function)

Differentiation of one function with respect to another function

Differentiate $\sin^2 x$ with respect to $\cos x$

Take $u = \sin^2 x$ and $v = \cos x$ then $\frac{du}{dv} = \frac{du}{dx} \times \frac{dx}{dv}$

Rules of logarithmic function

$\log_b mn = \log_b m + \log_b n$

$\log_b m/n = \log_b m - \log_b n$

$\log_b m^n = n\log_b m$

Change of base rule $\log_a b = \frac{\log_e b}{\log_e a}$

$\log e = 1$, $\log 1 = 0$, $e^{\log f(x)} = f(x)$

Example: If $y = x^{sinx} + sinx^{cosx}$, find $\frac{dy}{dx}$

Solution: $y = u+v$ then $\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ where $u = x^{sinx}$ and $v = sinx^{cosx}$

Parametric Form

If $x=f(t)$, $y = g(t)$ then $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$

Second Order Derivatives

Let $y=f(x)$, $\frac{dy}{dx} = f'(x) = y_1$ then $\frac{d^2y}{dx^2} = f''(x) = y_2$

If $x=f(t)$, $y = g(t)$ then $\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right) \times \frac{dt}{dx}$

Question Bank on Continuity & Differentiability

Basic Level Questions:

1. At what points is the function given by $f(x) = \frac{x+1}{(x-2)(x-3)}$ is continuous? **(Ans; 2,3)**
2. Find f(0), so that $f(x) = \frac{x}{1-\sqrt{1-x}}$ becomes continuous. **(Ans. F(0) = 2)**

3. If the function $f(x) = \frac{Sin\ 10x}{x}, x \neq 0$ is continuous at $x = 0$, find f(0). **(Ans. 10)**

4. Show that the exponential function e^x is continuous at each point of its domain.

5. Check the continuity of f(x) = 2x + 3 at x = 1.

6. Discuss the continuity of the function f(x) = |x| at x = 0.

7. Examine the continuity of the function at x = 2 if $f(x) = \begin{cases} 1 + x, if\ x \leq 2 \\ 5 - x, if\ x > 2 \end{cases}$.

8. Discuss the continuity of the function f at x = 0 if; $(x) = \begin{cases} 2x - 1, x < 0 \\ 2x + 1, x \geq 0 \end{cases}$.

9. Show that f(x) = 2x - |x| is continuous at x = 0.

10. Find the relation between a and b so that given function is continuous at x = 3, if $f(x) = \begin{cases} ax + 1, x \leq 3 \\ bx + 3, x > 3 \end{cases}$.

    (Ans. a = b + 2/3)

11. Show that the function f(x) = |x + 2| is continuous at every $x \in R$, but fails to be differentiate at x = 2.

Average Level Questions:

1. Find the value of a so that function $f(x)$ defined by $f(x) = \begin{cases} \frac{Sin^2 ax}{x^2}, x \neq 0 \\ 1, \quad x = 0 \end{cases}$ may be continuous at $x = 0$.

   (Ans. $a = \pm 1$)

2. Determine the value of k so that the function $f(x) = \begin{cases} kx^2, if\ x \leq 2 \\ 3, if\ x > 2 \end{cases}$ is continuous. (Ans. k = $\frac{3}{4}$)

3. Show that the function given by $f(x) = \begin{cases} x, if\ x \geq 1 \\ x^2, if\ x < 1 \end{cases}$ is continuous everywhere on R.

4. Show that the function: $f(x) = \begin{cases} x, \quad if\ x\ is\ an\ integer \\ 0, \quad if\ x\ is\ not\ an\ integer \end{cases}$ is discontinuous at each integral value of x.

5. Show that the function $f(x) = \begin{cases} \frac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1}, \ when\ x \neq 0 \\ 0, \ when\ x = 0 \end{cases}$ is discontinuous at x = 0.

6. Find the relationship between 'a' and 'b' so that the function 'f' defined by $f(x) = \begin{cases} ax + 1, \quad x \leq 3 \\ bx + 3, \quad x > 3 \end{cases}$ $is\ continuous\ at\ x = 3.$ (Ans. 3a – 3b = 2)

7. If the function $f(x) = \begin{cases} 3ax + b & if\ x > 1 \\ 11 & if\ x = 1 \\ 5ax - 2b & if\ x < 1 \end{cases}$ is continuous at $x = 1$, find a and b. (Ans. A = 3, b = 2)

8. Find all the points of discontinuity of the function given by $f(x) = \begin{cases} x + 2 & , x \leq 1 \\ x - 2, & 1 < x < 2 \\ 0 & , x \geq 2 \end{cases}$. (Ans. x = 1)

9. Find the value of a and b so that given function is continuous; $f(x) = \begin{cases} 5 & , x \leq 2 \\ ax + b, & 2 < x < 10 \\ 21 & , x \geq 10 \end{cases}$.

(Ans. a = 2, b = 1).

10. Locate the points of discontinuity of the function $f(x) = \begin{cases} \frac{x^4 - 16}{x - 2}, & if\ x \neq 2 \\ 16, & if\ x = 2 \end{cases}$ (Ans. x = 2,).

Above Average Level Questions:

1. If $f(x) = \begin{cases} \frac{x-5}{|x-5|} + a \\ a + b \\ \frac{x-5}{|x-5|} + b \end{cases}$ is continuous function. Find a and b. (Ans. a=1, b = -1)

2. Show that function f defined as follows, is continuous at x = 2, but not differentiable at x = 2.

$$f(x) = \begin{cases} 3x - 2, & 0 < x \leq 1 \\ 2x^2 - x, & 1 < x \leq 2. \\ 5x - 4 & , \quad x > 2 \end{cases}$$

3. Discuss the continuity of the function $f(x) = \begin{cases} |x| + 3, & x \leq -3 \\ -2x & , -3 < x < 3. \\ 6x + 2 & , x \geq 3 \end{cases}$ (Ans. Cont. at -3, discount. At 3)

4. Let$f(x) = \begin{cases} \frac{1 - sin^3 x}{3cos^2 x} \\ a \\ \frac{b(1 - \sin x)}{(\pi - 2x)^2} \end{cases}$ is continuous function at $= \frac{\pi}{2}$, find a and b. (Ans. a = ½, b = 4)

5. Find all the points of discontinuity of the function f defined by $f(x) = \begin{cases} x + 2 & , x \leq 1 \\ x - 2, & 1 < x < 2 \\ 0 & , x \geq 2 \end{cases}$

(Ans. Cont. at 2, discount. At -2)

6. For what value of 'k' is the following function continuous at x = 2? $f(x) = \begin{cases} 2x + 1 & , x < 2 \\ k & , x = 2 \\ 3x - 1 & , x > 2 \end{cases}$

(Ans. K = 5)

7. Discuss the continuity of the following function at x = 0. $f(x) = \begin{cases} \frac{x^4 + 2x^3 + x^2}{\tan^{-1} x} & , x \neq 0 \\ 0 & , \quad x = 0 \end{cases}$.

(Ans. Cont. at. x = 0)

8. Show that the function 'f' defined by $f = (x) = \left|1 - x + |x|\right|, x \in R$ is continuous.

9. Show that the function $f(x) = \begin{cases} x\ Sin\frac{1}{x}, if\ x \neq 0 \\ 0, \ if\ x = 0 \end{cases}$ is continuous at x = 0.

10. For what value of 'k', the function $f(x) = \begin{cases} \frac{\sqrt{5x+2}-\sqrt{4x+4}}{x-2}, if\ x \neq 2 \\ k,\ \ if\ x = 2 \end{cases}$ is continuous at x = 2.

(Ans. K = $\frac{1}{4\sqrt{3}}$)

Topic: Chain rule and differentiation of inverse trigonometric functions

Basic Level Questions:

Differentiate the following.

1. $y = (2x^3 - 7)^3$ — Ans. $18x^2 (2x^3 - 7)^2$
2. $y = (x^2 - 8x + 9)^4$ — Ans. $8(x-4) (x^2 - 8x + 9)^3$
3. $y = (x^4 - 9x^3 + 3x - 2)^2$ — Ans. $2(4x^3 - 27x^2 + 3)(x^4 - 9x^3 + 3x - 2)$
4. $y = e^{\sqrt{\cot x}}$ — Ans. $\frac{-cosec^2 x}{2\sqrt{cot\ x}} e^{\sqrt{\cot x}}$
5. $y = e^{x^2}$ — Ans. $2x e^{x^2}$
6. $y = \cos(x^2)$ — Ans. $-2x \sin(x^2)$
7. $y = \cos^2(x)$ — Ans. $-\sin 2x$
8. $y = \sin^{-1}(2x)$. — Ans. $\frac{2}{\sqrt{1-4x^2}}$
9. $y = \tan^{-1}(2 - x^2)$ — Ans. $\frac{-2x}{1+(2-x^2)^2}$
10. $y = \sin^{-1}(e^x)$ — Ans. $\frac{e^x}{\sqrt{1-e^{2x}}}$

Average Level Questions:

Differentiate the following.

1. $y = e^x + e^{2x} + e^{3x}$ — Ans. $e^x + 2e^{2x} + 3e^{3x}$
2. $y = \sin(e^x)$ — Ans. $e^x \cos(e^x)$
3. $y = xe^{2x}$ — Ans. $e^{2x}(2x + 1)$
4. $y = \cos(3x^2 + e^x)$ — Ans. $-(6x + e^x) \sin(3x^2 + e^x)$
5. $y = \tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$ — Ans. $\frac{1}{2}$
6. $y = 4\tan^{-1}(2x^4)$ — Ans. $\frac{32x^3}{1+4x^8}$
7. $y = (x^2 + 1)\sin^{-1}(3x)$
8. $y = e^{sin^{-1} x}$ — Ans. $\frac{1}{\sqrt{1-x^2}} e^{sin^{-1} x}$
9. $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ — Ans. $\frac{2}{1+x^2}$

10. y= $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$ Ans. $\frac{-2}{\sqrt{1-x^2}}$

Above Average Level Questions:

Differentiate the following.

1. $y = \sin^3(8x)$ Ans. $24\sin^2(8x).\cos(8x)$

2. $y = \sin(\sin(\sin(x)))$ Ans. $\cos(\sin(\sin(x)))\ (\cos(\sin(x)))\cos x$

3. $y = x \sin x \log x$ Ans. $\sin x \log x + x \cos x \log x + \sin x$

4. If y = $\tan^{-1}\left(\frac{5ax}{a^2-6x^2}\right)$ then show that $\frac{dy}{dx} = \frac{3a}{a^2+9x^2} + \frac{2a}{a^2+4x^2}$

5. If y = $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$+ $\sec^{-1}\left(\frac{1+x^2}{1-x^2}\right)$, then prove that $\frac{dy}{dx} = \frac{4}{1+x^2}$

Topic: Logarithmic and parametric differentiation

Basic Level Questions:

1. Find $\frac{dy}{dx}$ when $x = 2at^2$, $y = at^4$ Ans. t^2
2. Find $\frac{dy}{dx}$ when $x = a\cos^3 t$, $y = a\sin^3 t$ Ans. $-\tan t$
3. Find $\frac{dy}{dx}$ when $x = a(\Theta + \sin\Theta)$, $y = a(1 + \cos\Theta)$ Ans. $-\tan\frac{\theta}{2}$
4. Find $\frac{dy}{dx}$ when $x = e^t(\sin t + \cos t)$, $y = e^t(\sin t - \cos t)$ Ans. $\tan t$
5. Find $\frac{dy}{dx}$ if $y = e^{3\log x}$ Ans. $3x^2$
6. Find $\frac{dy}{dx}$ if $y = x^x$ Ans. $x^x(1 + \log x)$

Average Level Questions:

1. Find $\frac{dy}{dx}$ when $x = \frac{2t}{1-t^2}$, $y = \frac{2t}{1+t^2}$ Ans. $\left(\frac{1-t^2}{1+t^2}\right)^3$

2. Find $\frac{dy}{dx}$ when $y = a\cos^3\theta$, $x = b\sin^3\theta$ Ans. $-\frac{b}{a}\tan\theta$

3. Find $\frac{dy}{dx}$ when x = $e^t(1+\frac{1}{t})$ and y = $e^t(1-\frac{1}{t})$ Ans. $\frac{t^2-t+1}{t^2+t-1}$
4. Find $\frac{dy}{dx}$ when x = e^θ(2sin θ + sin2 θ) , y = e^θ(2cos θ + cos2 θ) Ans. $-\left(\frac{sin\theta+sin2\theta}{cos\theta+cos2\theta}\right)$
5. Find $\frac{dy}{dx}$ when $x^y y^x = a^b$ Ans. $\frac{-y}{x}\left(\frac{y+xlog\,y}{x+y\log x}\right)$

Above Average Level Questions:

1. If x = $\sqrt{a^{\sin^{-1}t}}$ and y = $\sqrt{a^{\cos^{-1}t}}$, show that $\frac{dy}{dx} = -\frac{y}{x}$
2. If x = sec Θ – cos Θ and y = $\sec^n\Theta - \cos^n\Theta$,
   then show that $(x^2 + 4)\left(\frac{dy}{dx}\right)^2 = n^2(y^2 + 4)$
3. If x = log t and y = $\frac{1}{t}$, prove that $y_2 + y_1 = 0$.
4. If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$
5. Find $\frac{dy}{dx}$ when $xy = e^{x-y}$ Ans. $\frac{y(x-1)}{x(y+1)}$

Topic: Second order derivatives

Basic Level Questions:

1. Find second order derivative of x^3.
2. If y= cot x, find $\frac{d^2y}{dx^2}$at $= \frac{\pi}{2}$.
3. If y=5cosx -3sinx , prove that$\frac{d^2y}{dx^2}$ +y =0.
4. If $\sqrt{x}+\sqrt{y}=\sqrt{a}$, find$\frac{d^2y}{dx^2}$ at x=a.
5. If y=$500e^{7x}+600e^{-7x}$, prove that $\frac{d^2y}{dx^2}$ = 49y.
6. If $\frac{x}{a}+\frac{y}{b}$ =1,find $\frac{d^2y}{dx^2}$.
7. If $x = a\,sin\,pt$and $y = b\,cos\,pt$, find the value of $\frac{d^2y}{dx^2}$ at x=0.
8. If $x = at^2, y = 2at$find$\frac{d^2y}{dx^2}$.
9. If $y = tanx$, prove that $\frac{d^2y}{dx^2}$= 2y$\frac{dy}{dx}$.
10. Find $\frac{d^2y}{dx^2}$ when $y = tanx + secx$

Average Level Questions:

.If y= $(\cos^{-1}x)^2$, prove that $(1-x^2)y_2 - xy_1=2$.

2.If y=log(x+$\sqrt{x^2+b^2}$) prove that $(x^2+b^2)y_2 + xy_1 = 0$.

3.If y=$e^{\tan^{-1}x}$ prove that $(1+x^2)\, y_2 + (2x-1)y_1 = 0$.

4. If x=$\frac{2at^2}{1+t}$ and y=$\frac{3at}{1+t}$find $\frac{d^2y}{dx^2}$.

5.If$xy = sinx$, prove that $\frac{d^2y}{dx^2}+\frac{2}{x}\frac{dy}{dx}$ + y =0.

6. If x=asin^{3}t , y=bcos^{3}t , find $\frac{d^2y}{dx^2}$ at t= $\frac{\pi}{4}$.

7.If y=3cos(logx) +4cos(logx) prove that

$x^2\frac{d^2y}{dx^2}$ +x$\frac{dy}{dx}$+ y =0

8. If y =e^x(sinx + cosx), prove that$\frac{d^2y}{dx^2}$ -2$\frac{dy}{dx}$ + 2y =0

9. If y=Ae^{mx}+ B e^{nx}, show that $\frac{d^2y}{dx^2}$- (m+n)$\frac{dy}{dx}$ mny=0.

10.Ife^y(x+1) =1 ,show that$\frac{d^2y}{dx^2}$ = $(\frac{dy}{dx})^2$

Above Average Level Questions:

1.If y= $(1+\sqrt{x^2-1})^m$, prove that $(x^2-1)\, y_2 + x\, y_1 = m^2 y$

2. y = $\frac{sin^{-1}x}{\sqrt{1-x^2}}$, show that $(1-x^2)\, y_2 - 3xy_1 - y = 0$.

3.If x= asin2t (1+cos2t) and y= bcos2t(1-cos2t) , find $\frac{d^2y}{dx^2}$ at t= $\frac{\pi}{4}$.

4.If y=x log$\left(\frac{a}{a+bx}\right)$, prove that $\frac{d^2y}{dx^2} = \frac{1}{X}\left(\frac{a}{a+bx}\right)^2$

5. If y= $e^{a\sin^{-1}x}$, prove that $(1-x^2)\, y_2 - xy_1 - a^2y = 0$

COMMON ERRORS COMMITTED BY STUDENTS IN CONTINUITY & DIFFERENTIABILITY

S. No.	Errors	Correction	Remarks
1.	$y = x^y + y^x$ $logy = logx^y + logy^x$	$y = x^y + y^x$ $y = u + v$ $u = x^y, log\, u = y\, log\, x$& $v = y^x, log\, v = x\, log\, y$	Proper application of logarithmic properties
2.	$\frac{d(log_e e)}{dx} = \frac{1}{e}$	$log_e e = 1$ $\frac{d(1)}{dx} = 0$	Proper application of logarithmic properties
3.	$\frac{d}{dx}(x^x) = x^x$	$\frac{d}{dx}(x^x) = x^x(1 + logx)$	Proper application of logarithmic properties
4.	$\frac{d}{dx}(x^x) = x.x^{x-1}$	$\frac{d}{dx}(x^x) = x^x(1 + logx)$	Proper application of logarithmic properties

5.	$\frac{d(\sin 2x)}{dx} = \cos 2x$	$\frac{d(\sin 2x)}{dx} = 2\cos 2x$	Application of chain rule
6.	$\frac{d}{dx}(a^x) = x.a^{x-1}$	$\frac{d}{dx}(a^x) = a^x log_e a$	Proper application of logarithmic properties
7.	$\frac{d}{dx}(a^x) = a^x$	$\frac{d}{dx}(a^x) = a^x log_e a$	Proper application of logarithmic properties
8.	$\frac{dy}{dx}$ **= sin t** $\frac{d^2y}{dx^2} = \cos t$	$\frac{dy}{dx}$ **= sin t** $\frac{d^2y}{dx^2} = \cos t.\frac{dt}{dx}$	Application of chain rule
9.	$If\ x = f(t) and\ y = g(t),$ $\frac{d^2y}{dx^2} = \frac{\frac{d^2y}{dt^2}}{\frac{d^2x}{dt^2}}$	$\frac{d^2y}{dx^2} = \frac{d(f(t))}{dt}.\frac{dt}{dx}$	Application of chain rule
10.	Differentiate $f(x) = (x-1)^{\frac{2}{3}}$on [0,2]. Answer: $f'(x) = \frac{2}{3}(x-1)^{\frac{-1}{3}}$	$f'(x) = \frac{2}{3}(x-1)^{\frac{-1}{3}}$, but Left hand derivative and right hand derivative at x = 1 are not equal. So at x = 1 it is not derivable and hence not differentiable in (0, 2)	Check LHD and RHD.
11.	$f(x) = \frac{1}{x}$ then $f'(x) = \frac{1\frac{d}{dx}x - x\frac{d}{dx}1}{x^2}$	$f'(x) = \frac{x\frac{d}{dx}1 - 1\frac{d}{dx}x}{x^2} = \frac{1}{x^2}$ $or\ f'(x) = \frac{d(x^{-1})}{dx} = -1\,x^{-2}$ $= \frac{-1}{x^2}$	Apply correct way of quotient rule.

Tips and techniques for scoring good marks

1) To show the function is continuous such as $\lim_{x\to a} f(x)$ is continuous at x = a then find $\lim_{x\to a} f(x)$ and f(a) if both are equal then it is continuous.
2) To show the function is continuous such as

   f(x) = $\begin{cases} f(x) & if\ x \neq a \\ g(x) & if\ x = a \end{cases}$ then find RHL ,LHL and f(a) if all are equal then f (x) is continuous.

3) To find the value of k if it is given that the function is continuous. Then find only RHL or LHL and f(a) and equate.
4) For logarithmic differentiation, students must be aware of all the rules of logarithmic & exponential.
5) To find second order derivative in parametric differentiation, proper use of chain rule should be followed.

<u>APPLICATION OF DERIVATIVES</u>

<u>Increasing / Decreasing Functions</u>

1) Let $x \,\&\, x'$ be any two points taken from an interval (a,b).

2) A real function $y = f(x)$ is ***increasing*** on the (a, b), if $f(x) < f(x')$ whenever $x < x'$.

3) The function is said to be ***decreasing*** on (a,b), if $f(x) > f(x')$, whenever $x < x'$.

<u>Increasing / decreasing test:</u>

The following result, called *Increasing/decreasing test,* is very useful in applications:

(i) If $f'(x) > 0$ for all $x \in (a,b)$, then f is increasing on (a,b)

(ii) If $f'(x) < 0$ for all $x \in (a,b)$, then f is decreasing on (a,b)

(iii) If $f'(x) = 0$ for all $x \in (a,b)$, then f is constant on (a,b).

*****:** A function f is said to be ***monotonic*** on an interval, if it is either increasing or decreasing on that interval.

5) A function $y = f(x)$ is said to have a ***critical point***at $x = c$, if any one of the following conditions is satisfied:

(a) $f'(c) = 0$ (b) $f'(c)$ is undefined, but $f(x)$ is continuous at $x = c$.

6) Let $y = f(x)$ be a given function. The points where $f'(x) = 0$ are called ***stationary points*** of the function. So, we can find the stationary points of a function $y = f(x)$ by solving the equation $f'(x) = 0$ for x.

<u>TANGENTS AND NORMALS</u>

1)For the curve y = f(x), $\frac{dy}{dx}$, represents the slope of the tangent to the curve

2) Slope of the tangent to the curve at (x_1, y_1) is $\frac{dy}{dx}]_{(x_1, y_1)}$.

3) Equation to the tangent at (**x_1, y_1**)to a curvey = f(x) with slope m is $y – y_1 = m(x – x_1)$

4) If m is the slope of the tangent to the curve y = f(x) at(**x_1, y_1**), then slope of the normal at (**x_1, y_1**) **is -1/m.**

5) Equation of normal at (**x_1, y_1**) is $y - y_1 = \frac{-1}{m}(x - x_1)$

Maxima & Minima

1)There are two types of extreme positions: ***local (relative)*** and ***global (absolute).***

2)A function $f(x)$ defined on an interval $[a,b]$ is said to have a ***local (or relative) maximum*** at a point $x=c$, if $f(c) \geq f(c+h)$ for all sufficiently small negative as well as positive values of h. The function is said to have a ***local (or relative) minimum*** at $x=c$, if $f(c) \leq f(c+h)$.

3)The point $x=c$, where $f'(c)=0$ or $f'(c)$ does not exist, is called a ***critical point*** of the function $f(x)$.

4)A maximum or a minimum value of a function is also termed as ***extremum*** or ***extreme value*** of the function.

5) Let f be function defined in the closed interval I. If there exist a point 'a' in the interval I such that $f(a) \geq f(x)$ for every x ϵ I, then the function is said to attain absolute maximum at x = a, and $f(a)$ is absolute maximum value.

6) Let f be function defined in the closed interval I. If there exist a point 'a' in the interval I such that $f(a) \leq f(x)$ for every x ϵ I, then the function is said to attain absolute minimum at x = a, and $f(a)$ is absolute minimum value.

7) To find the absolute maxima or minima in [a, b] we have to find out the value at the end point of interval [a, b] i.e. $f(a)$ and $f(b)$ along with local maxim or minima.

Test for maximum or minimum:

1) First Derivative Test : If a function $f(x)$ has either local maxima or minima at a point $x=c$, then either $f'(c)=0$ or $f'(c)$ does not exist, i.e, $x=c$ is a critical point of the function. Of course, there may be functions for which $f(c)$ is not a local extremum, even when $x=c$ is a critical point of the function.

2) If $f'(x)$ does not change its sign in the neighbourhood of 'x_1', then 'x_1' is neither point of local maxima nor local minima, then x_1 is called the point of inflexion.

- ***Second Derivative Test:***Let $f'(c)=0$ for a given function $f(x)$ defined on (a,b). Then
  (i) $f''(c)<0 \Rightarrow f(c)$ is a local maximum of $f(x)$

  (ii) $f''(c)>0 \Rightarrow f(c)$ is a local minimum of $f(x)$.

SUB TOPIC: INCREASING AND DECREASING

Level 1

Q1. Find the value of a for which the function f(x) = $x^2 - 2ax + 6$, x > 0 is strictly increasing. **(Ans. a $\leq$ 0)**

Q2. Write the interval for which the function f(x) = cos x, $0 \leq x \leq 2\pi$ is decreasing. **(Ans. $[0,\boldsymbol{\pi}]$)**

Q3. Write the interval for which the function f(x)= 1/x is strictly decreasing. **Ans.** $(-\infty,0)\cup(0,\infty)$.

Q4. If f (x) = ax + cos x is strictly increasing on R, find a. **Ans.** $\boldsymbol{a} \in (-\mathbf{1},\mathbf{1})$

Q5. Show that the function $\frac{\sin x}{x}$ is strictly decreasing in $(0,\frac{\pi}{2})$

Q6. Find the interval in which f(x)=$\frac{\log x}{x}$, $x \in (0,\infty)$ is increasing ? **Ans.** $\boldsymbol{x} \in (\mathbf{0},\boldsymbol{e})$

Q7. For which values of x, the functions y =$x^4 - \frac{4}{3}x^3$ is increasing? **Ans. f is increasing in [1, ∞)**

Q8. Find the least value of 'a' such that the function f(x) = $x^2 + ax + 1$ is strictly increasing on (1, 2).

Ans. $\boldsymbol{least\ value\ of\ a\ is -2}$)

Q9. Find the interval in which function is increasing. f(x) = $-2x^3 - 9x^2 - 12x + 1$ **(Ans.** $\boldsymbol{x \in (-\infty, -2)}$)

Q10. For what value of a, the function f(x) =a(x + sin x) + a ,is increasing or R. **{Ans. a∈[0,∞).}**

Level 2

Q 1 Show that f(x) = $x^3 - 6x^2 + 18x + 5$ is an increasing function for all x ∈ R

Q2 Write the interval in which the function f(x) = $x^9 + 3x^7 + 64$ is increasing. **[Ans. for all x ∈ R]**

Q3 Prove that the function f(x) = $x^2 - x + 1$ is neither increasing nor decreasing in [0, 1].

Q4 Find the intervals on which the function $\frac{x}{x^2+1}$ is decreasing. **Ans.** $(-\infty, -\mathbf{1}) \cup (\mathbf{1}, \infty)$

Q5 Find the intervals in which the function f(x) =$sin^2x\ in[0, \pi] is\ increasing\ or\ decreasing$

Ans :$\boldsymbol{inc\left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right) and\ dec\left(\frac{\pi}{2}, \pi\right) \cup \left(\frac{3\pi}{2}, 2\pi\right)}$

Q7 Find the intervals in which the function f(x) = $(x + 1)^3 (x - 3)^3$ is strictly increasing or strictly decreasing.

Q8 Find the interval in which the function f(x)=$5x^{\frac{3}{2}} - 3x^{\frac{5}{2}}, x > 0$ is strictly decreasing.

Ans: Inc (0,1) and dec (1,∞)

Q9 Prove that the function f(x) =$\frac{x^3}{3} - x^2 + 9x$ is strictly increasing. Hence find the minimum value of f(x)

Q10 Find the intervals in which the function f(x) = $x^3 - 12x^2 + 36x + 17$ is

(a) Increasing (b) Decreasing. **Ans :**$\boldsymbol{inc(-\infty, 2) \cup (6, \infty) and\ \ dec(2, 6)}$

Level 3

Q1 Find the intervals in which the function f(x) = $\sin^4x + \cos^4x$, $0 \le x \le \frac{\pi}{2}$ is increasing or decreasing. **Ans:** $\boldsymbol{inc.\left(\frac{\pi}{4}, \frac{\pi}{2}\right) dec\left(0, \frac{\pi}{4}\right)}$

Q2 Find the intervals in which the function f(x) = log (1 + x) $- \frac{x}{1+x}$ $where\ x > -1$is increasing or decreasing **Ans:** $\boldsymbol{inc}$ **(0,∞)** $\boldsymbol{dec(-1, 0)}$

Q3 Show that the function f(x) = $\tan^{-1}$ (sin x + cos x), is strictly increasing on the interval $\left(0, \frac{\pi}{4}\right)$

Q 4 Find the interval in which the function f given by f(x) = sin x - cos x, 0 <x < 2π is increasing or decreasing. **Ans :**$\boldsymbol{inc\left(0, \frac{3\pi}{4}\right) \cup \left(\frac{7\pi}{4}, 2\pi\right) and\ dec\left(\frac{3\pi}{4}, \frac{7\pi}{4}\right)}$

Q5 Prove that the function y = $\frac{4\sin\theta}{2+\cos\theta} - \theta$is an increasing function of θin [0, $\frac{\pi}{2}$]

Q6 Find the interval of monotonicity of the function f(x) = $2x^2 - \log x$, x ≠ 0

Ans :$\boldsymbol{inc\left(\frac{-1}{2}, 0\right) \cup \left(\frac{1}{2}, \infty\right) and\ dec\left(-\infty, \frac{-1}{2}\right) \cup \left(0, \frac{1}{2}\right)}$

Q7 Find the intervals for which the function f(x) =log(2 +x) $- \frac{2x}{2+x}$ is increasing or decreasing **Ans:** $\boldsymbol{inc.}$**(2,∞)**, $\boldsymbol{dec(-2, 2)}$

Q8 Find the interval in which the given function is increasing or decreasing:

f(x)=$\frac{4\,sinx - 2x - x\,cosx}{2+\cos x}$ Ans: $inc\left(0, \frac{\pi}{2}\right) \cup \left(\frac{3\pi}{2}, 2\pi\right)\ dec\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

Q9 Show that the function cos (2x+$\frac{\pi}{4}$) is strictly increasing on$\left(\frac{3\pi}{8}, \frac{7\pi}{8}\right)$

Q10 Find the sub-interval of the interval (0, π/2) in which the function f(x) = sin 3x is increasing. **Ans.** $\boldsymbol{x \in [0, \frac{\pi}{6}]}$

Sub Topic :Tangents and Normal

Level 1

Q1 Find the slope of the tangent to the curve $y = 3x^4 - 4x$ at $x = 4$. **(slope of tangent = 764)**

Q2 Find the slope of the tangent to the curve $y = x^3 - x + 1$ at the point whose x –coordinate is 2. **(slope of tangent = 11)**

Q3 Find the equations of tangent and normal to the curve $y^2 = \frac{x^2}{4-x}$ at (3, - 3)
(Ans. 5x +2y = 9: 2x – 5y = 21)

Q4 Find the slope of the normal at the point (am^3, am^2) to the curve $ax^2 = y^3$. (Ans. -3m/2)

Q5 At what points on the curve $x^2 + y^2 - 2x - 4y + 1 = 0$, the tangents are parallel to x – axis?
[Ans. (1, 0) & (1, 4)]

Q6 Find the equations of normal lines to the curve $y = x^3 - 3x$ which are parallel to the line $x + 9y = 14$. **(Ans. x +9y = ±20)** .

Level 2.

Q 1 Find the equation of tangents to the curve $y = (x^3 - 1)(x - 2)$ where the curve cuts x- axis.
(Ans. y = -3x + 3: y = 7x -14)

Q 2 Find the points on the curve $4x^2 + 9y^2 = 1$, where the tangents are perpendicular to the line
$2y + x = 0$. **[Ans. $\left(\frac{\pm 3}{2\sqrt{10}}, \frac{\mp 1}{3\sqrt{10}}\right)$]**

Q 3 Prove that the curves $y^2 = 4ax$ and $xy = c^2$ cut at right angles, if $c^4 = 32a^4$.

Q 4 Show that the curves $x = y^2$ and $xy = k$ cut orthogonally if $8k^2 = 1$.

Q 5 Show that the curves $2x = y^2$ and $2xy = k$ cut at right angles if $k^2 = 8$.

Level 3

Q1 At what point(s) on the curve $y = x^2$ does the tangent make an angle of 45° wit x-axis
[Ans. $(\frac{1}{2}, \frac{1}{4}1)$

Q2 Show that the equation of tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at (x_1, y_1) is $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$.

Q3 Prove that the curves $xy = 4$ and $x^2 + y^2 = 8$ touch each other.

Q4 Find the equation of tangent to the curve $x = \sin 3t$, $y = \cos 2t$ at $t = \pi/4$.
[Ans. $2\sqrt{2}x - 3y = 2$]

Q7 For the curve $y = 4x^3 - 2x^5$, find all the points at which the tangents passes through the origin.
[Ans. (0, 0), (1, 2), (-1, -2)

Q8 If the tangent to the curve $y = x^3 + ax + b$ at point(1, - 6)is parallel to the line $y - x = 5$. Find the value of a and b. **(a = -2, b = -5)**

Sub Topic :Maxima and Mininma

Level 1

Q1 What is the maximum and minimum value of the function f(x) = x?
(Ans: f(x) has no extreme value)

Q2 At what value of x is the local maxima for the function $f(x) = x^3 - 3x + 3$? **(Ans: x = 1)**

Q3 What is the minimum value of $x^2 + (250/x)(x>0)$? **(Ans: 75)**

Q4 Maximum value of f(x) = sinxcosx is _______ **(Ans: 1/2)**

Q5 Find the point on the curve $y^2 = 4x$ which is the nearest to the point (2,-8). **(Ans. (4, -4))**

Q6 Find the maximum and minimum value of the function f(x) = 3-2sinx, in **(Ans. (5 & 1))**

Q7 Find two positive numbers whose sum is 24 and whose product is maximum. **(Ans. (12,12))**

Q8 Show that of all the rectangle of given area, the square has the smallest perimeter.

Q9 Show that the function $f(x) = x^3 + x^2 + x + 1$ has neither a maximum value nor a minimum value.

Q10 A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is 10m. Find the diameter of the window to get maximum light through the whole opening. **(Ans: d = $\frac{20}{\pi+4}$)**

Level 2

Q1 Check whether the function f(x) = $2x^3-6x^2+6x+5$ has a local maximum or local minimum at

x = 1? **(Ans: Neither minima or maxima at x = 1)**

Q2 Let f(x) = { |x| for 0<|x|<=1 then f has a _____ at x=0 1 for x = 0. **(Ans: Local Maxima)**

Q3 At x = 0 the function f(x) = x^3 has a ______ **(Ans: Point of inflection)**

Q4 What is the maximum value of f(x) = $1/(4x^2+2x+1)$, xεR ? **(Ans: 4/3)**

Q5 **Prove that the** area of a right angled triangle of given hypotenuse is maximum when the triangle is isosceles.

Q6 Find the area of the greatest isosceles triangle that can be inscribed in a given ellipse $x^2/a^2 + y^2/b^2=1$ with its vertex coinciding with one extremity of the major axis. **($\frac{3\sqrt{3}}{4}ab$ sq unit)**

Q7 Show that the height of a closed right circular cylinder of given surface and maximum volume is equal to the diameter of the base.

Q8 A piece of wire 28 units long is cut into two pieces. One piece is bent into the shape of a circle and other into the shape of a square. How the wire should be cut so that the combined area of the two figures is as small as possible. **(Ans. $\frac{112}{4+\pi}$ & $\frac{28\pi}{4+\pi}$)**

Q9 A farmer wants to construct a circular well and a square garden in his field. He wants to keep sum of their perimeters fixed. **Then prove that** the sum of their areas is least when the side of square garden is double the radius of the circular well. Do you think good planning can save energy, time and money?

Q10 If length of 3 sides of trapezium other than base are equal to 10cm each, then find the area of trapezium when it is maximum? **(ans $75\sqrt{3}cm^2$)**

Level 3

Q1 A manufacturer can sell x items at a price of Rs 5 –(x/100) each. The cost price of x pens is Rs (x/5) + 500. What is the number of items, the manufacturer should sell to earn maximum profit? **(Ans: 240)**

Q2 Find the maximum and minimum value of the function without using derivative f(x) = -|x+2|+3. **(Ans: Max. =3, Min does not exists)**

Q3 Show that $y=e^x$ has no local maxima or local minima.

Q4 A jet of an enemy is flying along the curve $y=x^2+2$. A soldier is placed at the point (3,2). What is the nearest distance between the soldier and the jet. **(Ans.$\sqrt{5}$)**

Q5 Find the altitude of a right circular cone of maximum curved surface which can be inscribed in a sphere of radius r. **(Ans.4r/3)**

Q6 Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of sphere.

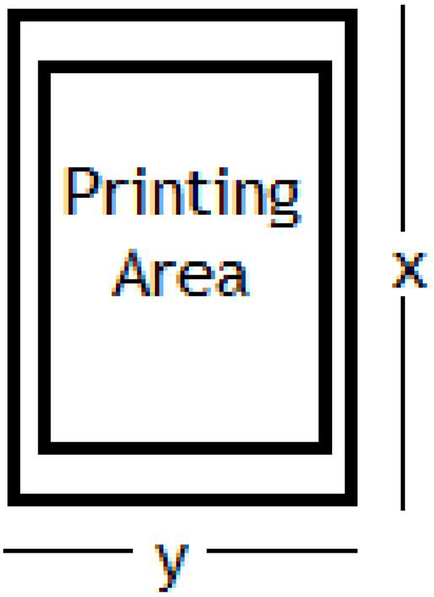
Q7 Show that the semi vertical angle of the cone of the maximum volume & of given slant height is

$tan^{-1}\sqrt{2}$.

Q 8 Profit function of a company is given as P(x) =(24x/5) –(x^2/100) – 500 where x is the number of units produced. What is the maximum profit of the company? Company feels its social responsibility and decided to donate 10% of his profit for the orphanage. What is amount contributed by the company for the charity? **(Max profit is Rs. 76 at x = 240)**

Case Study based Questions (Chapter-6)

Q01. Following is the pictorial description for a page.



The total area of the page is 150 cm^2. The combined width of the margin at the top and bottom
is 3 cm and the side 2 cm.
Using the information given above, answer the following :

(i) The relation between x and y is given by

(a) $(x - 3)y = 150$

(b) $xy = 150$

(c) $x(y - 2) = 150$

(d) $(x - 2)(y - 3) = 150$

(ii) The area of page where printing can be done, is given by

(a) xy

(b) $(x + 3)(y + 2)$

(c) $(x - 3)(y - 2)$

(d) $(x - 3)(y + 2)$

(iii) The area of the printable region of the page, in terms of x, is

(a) $156 + 2x + \frac{450}{x}$

(b) $156 - 2x + 3\left(\frac{150}{x}\right)$

(c) $156 - 2x - 15\left(\frac{3}{x}\right)$

(d) $156 - 2x - 3\left(\frac{150}{x}\right)$

iv) For what value of 'x', the printable area of the page is maximum?

(a) 15 cm

(b) 10 cm

(c) 12 cm

(d) 15 units

(v) What should be dimension of the page so that it has maximum area to be printed?

(a) Length =1 cm, width =15 cm

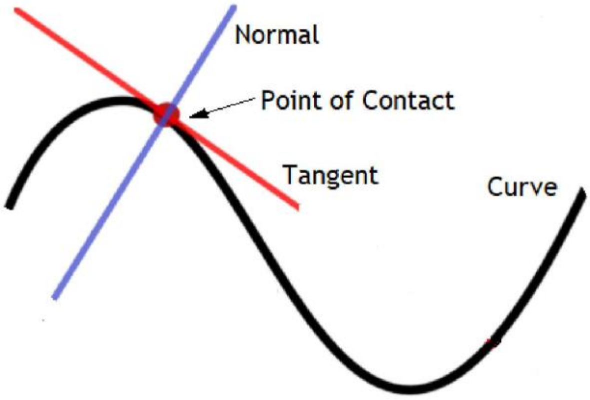
(b) Length =15 cm, width =10 cm

(c) Length =15 cm, width = 12 cm

(d) Length =150 cm, width =1 cm

Q02. There is a toy in the form of a curve, whose equation is given as y = f (x) .

To make it a look more fancy, some straight sticks are crafted over it.



A student wishes to learn about the tangent and normal to the curve.

His teacher explained him that the Tangent to the curve means a line which touches the curve at a point - this point is called the 'point of contact'. The teacher further adds that if we draw a line perpendicular to the tangent at the point of contact - then this perpendicular line is called a Normal to the curve.

Using derivatives, answer the following with reference to the curve

$f\ (x) = \ (x-3)^2$.

(i) A student wants to draw a straight line which touches the parabolic curve given above at a specific point *say* (2, 1). The equation of this line is

(a) $2x + □y + □5 = □0$

(b) $x + □2y = □5$

(c) $2x + \ y = □0$

(d) $2x + \ y = □5$

(ii) Slope of the tangent to the parabolic curve given above at (3, 0) is

(a) 0

(b) 1

(c) 2

(d) –1

(iii) The normal to the curve y ▯$= (x-3)^2$ at (3, 0) is

(a) parallel to x-axis

(b) parallel to y-axis

(c) perpendicular to y-axis

(d) can not be determined

(iv) The point on the given curve ▯$= (x-3)^2$, where the tangent is parallel to the line joining the points (4, 1) and (3, 0) is

(a) (1, 7)

(b) $\left(\frac{7}{2}, \frac{1}{4}\right)$

(c) (−7, 1)

(d) (7, 4)

(v) The product of slopes of tangent and normal to the given curve, at (2, 1) is

(a) 0

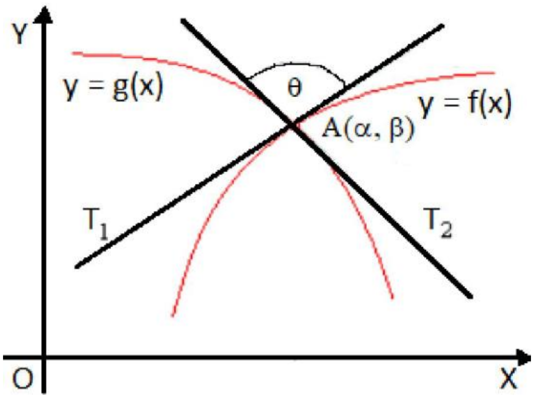
(b) 1

(c) –1

(d) 2

Q 03. Assuming that two ships follow the path of curves $y = f(x)$ and $y = g(x)$ .Let the two curves intersect each other at a point $A(\alpha, \beta)$.

When we draw tangents to these curves at the point of intersection, then ‘the angle between these tangents’ is called the ‘angle between the two curves’.



Using the information given above, answer the following with reference to the curves $y = x^2$ and $x = y^2$:

(i) The point (s) of intersection for the above curves is (are)

(a) (0, 0), (1, ±1) (b) (0, 0), (1, 1) (c) (0, –1), (1, 0) (d) (1, 0), (0, 1)

(ii) What are the numbers of points at which the given two curves intersect?

(a) 2 (b) 1 (c) 3 (d) 0

(iii) The slope of curve $x = y^2$ at the point of intersection of both the given curves, is

(a) $\frac{1}{2}, -\frac{1}{2}, \frac{1}{0}$

(b) $\frac{1}{2}, 0$

(c) $-\frac{1}{2}, \frac{1}{0}$ (Not defined)

(d) $\frac{1}{2}, \frac{1}{0}$ (Not defined)

(iv) The slope of tangent to the curve $y = x^2$ at the point of intersection of both the given curves, is

(a) 0, 2

(b) 2, –2

(c) 0, –1

(d) 2, –2, 0

(v) The angle of intersection of both the curves is

(a) $\pi, \tan^{-1}\frac{3}{4}$

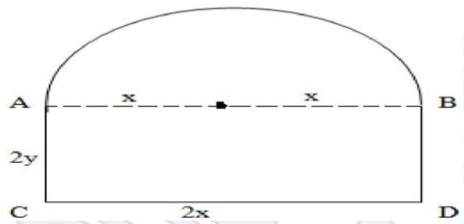
(b) $\frac{\pi}{2}, \tan^{-1}\frac{4}{3}$

(c) $\frac{\pi}{2}, \tan^{-1}\frac{3}{4}$

(d) $-\frac{\pi}{2}, \tan^{-1}\frac{3}{4}$

Q 04. Mr Shashi, who is an architect, designs a building for a small company. The design of window on the ground floor is proposed to be different than other floors. The window

is in the shape of a rectangle which is surmounted by a semi-circular opening. This window is having a perimeter of 10 m as shown below :



Based on the above information answer the following :

(i) If 2x and 2y represents the length and breadth of the rectangular portion of the windows, then the relation between the variables is:

(a) $4y - 2x = 10 - \pi$

(b) $4y = 10 - (2 - \pi)\, x$

(c) $4y = 10 - (2 + \pi)\, x$

(d) $4y - 2x = 10 + \pi$

(ii) The combined area (A) of the rectangular region and semi-circular region of the window expressed as a function of x is:

(a) A= $10x + \left(2 + \frac{1}{2}\pi\right) x^2$

(b) A= $10x - \left(2 + \frac{1}{2}\pi\right) x^2$

(c) A= $10x + \left(2 - \frac{1}{2}\pi\right) x^2$

(d) A= $4xy + \frac{1}{2}\pi x^2$, where $y = \frac{5}{2} + \frac{1}{4}(2 + \pi)\, x$

(iii) The maximum value of area A, of the whole window is

(a) $\frac{50}{\pi - 4}$

(b) $\frac{50}{4 + \pi}$

(c) $\frac{100}{4 + \pi}$

(d) $\frac{50}{4 - \pi}$

(iv) The owner of this small company is interested in maximizing the area of the whole window so that maximum light input is possible.

For this to happen, the length of rectangular portion of the window should be

(a) $\frac{20}{4 + \pi}$

(b) $\frac{10}{4 + \pi}$

(c) $\frac{4}{10 + \pi}$

(d) $\frac{100}{4 + \pi}$

(v) In order to get the maximum light input through the whole window, the area (in sq. m) of only semi-circular opening of the window is

(a) $\frac{100}{(4+\pi)^2}$

(b) $\frac{50\,\pi}{4+\pi}$

(c) $\frac{50\,\pi}{(4+\pi)^2}$

(d) Same as the area of rectangular portion of the window

Answer:

(i) c
(ii) b
(iii) b
(iv) a
(v) b

Tips and Techniques to score high

General

1. Student must know about the Time management so that He/She can attempt the entire question paper.
2. Student must not write the unwanted steps in a solution for time saving purpose.
3. Student must utilize the reading time in planning to attempt the question paper.
4. 01 Mark questions should be dealt with straight answer without the solving steps.
5. Question carrying 06 marks may be attempted first.

Sub Topic related

6. In case of finding the equation of Tangent and Normal it is better to draw the rough sketches of the tangent and Normal to the curve.
7. In maxima minima problems student at least should write the respective mensuration formula and should draw the appropriate figure of it.
8. In case of Increasing Decreasing function student should draw the number line to represent intervals in which the function is increasing or decreasing.
9. While differentiating a function w.r.t. any variable for finding increasing or decreasing domain of the function should always kept in mind. i.e. for log function ,inverse function etc.

10. In maxima minima problem always write the inter relation of the variables contained in the maximizing/ minimizing problems.

Common errors and mistakes committed by Students during exam

Application of Derivatives

1) Increasing and decreasing function

1) Strictly increasing in open interval but student generally writes in close interval i.e [1,2] instead of (1,2).

2) Instead of finding the sign of $f'(x)$ students find the sign of $f(x)$ in between the critical points.

2)Tangent and Normal

1) Student usually finds slope of tangent and normal to the curve at a general point instead of Slope of the tangent and normal at a given point for finding the equation of the tangent and normal at a given point.
2) Children get confused while finding the tangent to the curve at a point on the curve and tangent from an external point to the curve.

3)Maxima and Minima

1) After finding the critical point student generally do not finds the sign. Of second derivative to decide the Maxima or Minima.
2) Sometimes they differentiates the constants also w.r.t. the independent variable. i.e.
   V=πr^2h (h is a constant)
   dv/dr = 2πr

INTEGRATION

INTRODUCTION

IF f(x) is derivative of function g(x), then g(x) is known as antiderivative or integral of f(x)

i.e., $\frac{d}{dx}$**(g(x)) = f(x)** $\Leftrightarrow$ $\int f(x)dx = g(x)$

STANDARD SET OF FORMULAS

*** Where c is an arbitrary constant.**

1. $\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$ **(n $\neq$ -1)**

2. $\int dx = x + c$

3. $\int \frac{1}{x} dx = \log|x| + c$

4. $\int \cos x \, dx = \sin x + c$

5. $\int \sin x \, dx = -\cos x + c$

6. $\int \sec^2 x \, dx = \tan x + c$

7. $\int cosec^2 x \, dx = -\cot x + c$

8. $\int \sec x \tan x \, dx = \sec x + c$

9. $\int cosec\, x \cot x \, dx = -cosec\, x + c$

10. $\int e^x \, dx = e^x + c$

11. $\int \tan x \, dx = \log|\sec x| + c$

12. $\int \cot x \, dx = \log|\sin x| + c$

13. $\int \sec x \, dx = \log|\sec x + \tan x| + c$

14. $\int cosec\, x \, dx = \log|cosec\, x - \cot x| + c$

15. $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + c$

16. $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

17. $\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1} x + c$

18. $\int a^x \, dx = \frac{a^x}{\log a} + c$

19. $\int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} + c$

INTEGRALS OF LINEAR FUNCTIONS

1. $\int (ax+b)^n \, dx \quad = \frac{(ax+b)^{n+1}}{(n+1)a} + c$

2. $\int \frac{1}{ax+b} \, dx \quad = \frac{\log(ax+b)}{a} + c$

3. $\int \sin(ax+b)\,dx \quad = \frac{-\cos(ax+b)}{a} + c$

In the same way if ax +b comes in the place of x, in the standard set of formulas, then divide the integral by a

SPECIAL INTEGRALS

1. $\int \frac{1}{x^2-a^2} \, dx \quad = \frac{1}{2a} \log\left|\frac{x-a}{x+a}\right| + c$

2. $\int \frac{1}{a^2-x^2} \, dx \quad = \frac{1}{2a} \log\left|\frac{a+x}{a-x}\right| + c$

3. $\int \frac{1}{x^2+a^2} \, dx \quad = \frac{1}{a} \tan^{-1}\frac{x}{a} + c$

4. $\int \frac{1}{\sqrt{x^2-a^2}} \, dx \quad = \log\left| x + \sqrt{x^2-a^2} \right| + c$

5. $\int \frac{1}{\sqrt{x^2+a^2}} \, dx \quad = \log\left| x + \sqrt{x^2+a^2} \right| + c$

6. $\int \frac{1}{\sqrt{a^2-x^2}} \, dx \quad = \sin^{-1}\left(\frac{x}{a}\right) + c$

7. $\int \sqrt{x^2+a^2} \, dx \quad = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2} \log\left| x + \sqrt{x^2+a^2} \right| + c$

8. $\int \sqrt{x^2-a^2} \, dx \quad = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2} \log\left| x + \sqrt{x^2-a^2} \right| + c$

9. $\int \sqrt{a^2-x^2} \, dx \quad = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2} \sin^{-1}\left(\frac{x}{a}\right) + c$

INTEGRATION BY PARTS

1. $\int u.v \, dx \quad = u.\int v\,dx - \int\left(\frac{du}{dx}\int v\,dx\right) dx$

OR

The integral of product of two functions = (first function) x integral of the second function – integral of [(differential coefficient of the first function) × (integral of the second function)]

We can choose first and second function according to I L A T E where I → inverse trigonometric function $L \to \textit{logarithmic function}$, $\mathrm{A} \to \textit{algebraic function}$

$\mathrm{T} \to \textit{trigonometric function} \quad \mathrm{E} \to \textit{exponential function}$

2. $\int e^x(f(x) + f'(x))\,dx = e^x f(x) + c$

Working Rule for different types of integrals

1. Integration of trigonometric function

Working Rule

(a) Express the given integrand as the algebraic sum of the functions of the following forms

(i) Sin kα , (ii) cos kα ,(iii) tan kα , (iv) cot kα , (v) sec kα ,(vi) cosec kα, (vii) sec^2 kα , (viii) cosec^2 kα , (ix) sec kα tan kα (x) cosec kα cot kα

For this use the following formulae whichever applicable

(i) $sin^2 x = \frac{1-cos\,2x}{2}$ (ii) $cos^2 x = \frac{1+cos\,2x}{2}$

(iii) $sin^3 x = \frac{3\,sin\,x - sin\,3x}{4}$ (iv) $cos^3 x = \frac{3\,cos\,x + cos\,3x}{4}$

(v) $\tan^2 x = \sec^2 x - 1$ (vi) $\cot^2 x = \text{cosec}^2 x - 1$

(vii) $2\sin x \sin y = \cos(x - y) - \cos(x + y)$

(viii) $2\cos x \cos y = \cos(x + y) + \cos(x - y)$

(ix) $2\sin x \cos y = \sin(x + y) + \sin(x - y)$

(x) $2\cos x \sin y = \sin(x + y) - \sin(x - y)$

2. Integration by substitution

(a) Consider I = $\int f(x)dx$

Put x = g(t) so that $\frac{dx}{dt} = g'(t)$

We write dx= $g'(t)dt$. Thus I= $\int f(x)dx = \int f(g(t)\ g'(t)\ dt$

(b) When the integrand is the product of two functions and one of them is a function g (x) and the other is k $g'(x)$, where k is a constant then Put g (x) = t

3. Integration of the types $\int \frac{dx}{ax^2+bx+c}$, $\int \frac{dx}{\sqrt{ax^2+bx+c}}$

In these forms change $ax^2 + bx + c$ in the form $A^2 + X^2$, $X^2 - A^2$, or $A^2 - X^2$

Where X is of the form x +k and A is a constant (by completing square method)

Then integral can be find by using any of the special integral formulae.

4. Integration of the types $\int \frac{px+q}{ax^2+bx+c}\,dx$, $\int \frac{px+q}{\sqrt{ax^2+bx+c}}\,dx$

In these forms split the linear px +q = $\lambda \frac{d}{dx}(ax^2 + bx + c) + \mu$

Then divide the integral into two integrals

The first integral can be find out by method of substitution and the second integral by completing square method as explained in 3

i,e., to evaluate $\int \frac{px+q}{\sqrt{ax^2+bx+c}}\, dx = \int \frac{\lambda\ (2ax+b\ )+\mu}{ax^2+\ bx+c}$ dx

$$= \lambda \int \frac{2ax+b}{ax^2+\ bx+c}\ dx + \mu \int \frac{dx}{ax^2+\ bx+c}$$

↓ ↓

Find by substitution method + by completing square method

5. **Integration of rational functions**

In the case of rational function, if the degree of the numerator is equal or greater than degree of the denominator , then first divide the numerator by denominator and write it as

$\frac{Numerator}{Denominator} = Quotient + \frac{Remainder}{Denominatior}$, then integrate

6. **Integration by partial fractions**

Integration by partial fraction is applicable for rational functions. There first we must check thatdegree of the numerator is less than degree of the denominator, if not, divide the numerator by denominator and write as $\frac{Numerator}{Denominator} = Quotient + \frac{Remainder}{Denominatior}$ and proceed for partial fraction of $\frac{Remainder}{Denominatior}$

Sl. No.	Form of the rational functions	Form of the rational functions
1	$\frac{px+q}{(x-a)(x-b\ )}$	$\frac{A}{x-a}+\frac{B}{x-b}$
2	$\frac{px+q}{(x-a)^2}$	$\frac{A}{x-a}+\frac{B}{(x-a)^2}$
3.	$\frac{px^2+qx+r}{(x-a\ )(x-b\ )x-c\ )}$	$\frac{A}{x-a}+\frac{B}{x-b}+\frac{C}{x-c}$
4	$\frac{px^2+qx+r}{(x-a\ )^2(x-b\ )}$	$\frac{A}{x-a}+\frac{B}{(x-a\ )^2}+\frac{C}{x-b}$
5	$\frac{px^2+qx+r}{(x-a\ )(x^2+\ bx+c\ )}$ Where x^2 + bx + c cannot be factorized further	$\frac{A}{x-a}+\frac{Bx+C}{x^2+\ bx+c}$

DEFINITE INTEGRATION

Working Rule for different types of definite integrals

1. Problems in which integral can be found by direct use of standard formula or by transformation method

Working Rule

(i). Find the indefinite integral without constant c

(ii). Then put the upper limit b in the place of x and lower limit a in the place of x and subtract the second value from the first. This will be the required definite integral.

2. Problems in which definite integral can be found by substitution method

Working Rule

When definite integral is to be found by substitution then change the lower and upper limits of integration. If substitution is z = ϕ(x) and lower limit integration is a and upper limit is b Then new lower and upper limits will be ϕ(a) and ϕ(b) respectively.

Properties of Definite integrals

1. $\int_a^b f(x)dx = \int_a^b f(t)dt$

2. $\int_a^b f(x)dx = \int_b^a f(x)dx$. In particular, $\int_a^a f(x)dx = 0$

3. $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$, a < c < b

4. $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

5. $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

6. $\int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(2a-x)dx$

7. $\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx$, if $f(2a-x) = f(x)$ and

   =0 , if $f(2a-x) = -f(x)$

8. (i) $\int_{-a}^a f(x)dx = 2\int_0^a f(x)dx$, if f is an even function, i.e., if $f(-x) = f(x)$

   (ii) $\int_{-a}^a f(x)dx = 0$, if f is an odd function, i.e., if $f(-x) = -f(x)$

Problem based on property

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx, \quad a < c < b$$

Working Rule: This property should be used if the integrand is different in different parts of the interval [a,b] in which function is to be integrand. This property should also be used when the

integrand (function which is to be integrated) is under modulus sign or is discontinuous at some points in interval [a,b]. In case integrand contains modulus then equate the functions whose modulus occur to zero and from this find those values of x which lie between lower and upper limits of definite integration and then use the property.

<u>Problem based on property</u>

$$\int_0^a f(x)dx = \int_0^a f(a-x)dx$$

Working Rule

Let $\quad$ **I =** $\int_0^a f(x)dx$

Then $\quad$ **I =** $\int_0^a f(a-x)dx$

(1) + (2) => $\quad$ **2I =** $\int_0^a f(x)dx + \int_0^a f(a-x)dx$

I = $\frac{1}{2}\int_0^a \{f(x) + f(a-x)\}dx$

This property should be used when $f(x) + f(a-x)$ becomes an integral function of x.

Problem based on property

$\int_{-a}^a f(x)dx = 0$**, if $f(x)$ is an odd function and** $\int_{-a}^a f(x)dx = 2\int_0^a f(x)dx$**, if $f(x)$ is an even function.**

Working Rule

This property should be used only when limits are equal and opposite and the function which is to be integrated is either odd/ even.

<u>SOLVED PROBLEMS</u>

Evaluate the following integrals

1. $\int \frac{(1+\log X)^2}{X}\, dx$

Solution : $\quad$ **put 1+log x = t**

$$\frac{1}{x}\, dx = dt$$

$\int \frac{(1+\log X)^2}{X}\, dx$ = $\int t^2\, dt$

= $\frac{t^3}{3} + c$

= $\frac{(1+\log x)^3}{3} + c$

2. $\int \frac{e^x}{\sqrt{5-4e^x-e^{2x}}}\, dx$

Solution

Put $e^x = t$ $then\ e^x\,dx = dt$

$$\int \frac{e^x}{\sqrt{5-4e^x-e^{2x}}}\,dx = \int \frac{dt}{\sqrt{5-4t-t^2}}$$

$$= \int \frac{dt}{\sqrt{-(t^2+4t-5)}}$$

$$= \int \frac{dt}{\sqrt{-(t^2+4t+4-4-5)}}$$

$$= \int \frac{dt}{\sqrt{-\{(t+2)^2-9\}}}$$

$$= \int \frac{dt}{\sqrt{3^2-(t+2)^2}}$$

$$= \sin^{-1}\frac{t+2}{3} + C = \sin^{-1}\left(\frac{e^x+2}{3}\right) + C$$

3. $\int \frac{5x+3}{\sqrt{x^2+4x+10}}\,dx$

Solution

5x + 3 = A (2x + 4) + B => A = $\frac{5}{2}$ $and\ B = -7$

$$\int \frac{5x+3}{\sqrt{x^2+4x+10}}\,dx = \int \frac{\frac{5}{2}(2x+4)-7}{\sqrt{x^2+4x+10}}\,dx$$

$$= \int \frac{\frac{5}{2}(2x+4)}{\sqrt{x^2+4x+10}}\,dx + \int \frac{7}{\sqrt{x^2+4x+10}}\,dx$$

$$= \frac{5}{2}\int \frac{(2x+4)}{\sqrt{x^2+4x+10}}\,dx + 7\int \frac{1}{\sqrt{x^2+4x+10}}\,dx$$

$$= \frac{5}{2}\int \frac{dt}{\sqrt{t}} + 7\int \frac{1}{\sqrt{x^2+4x+4-4+10}}\,dx$$

$$= \frac{5}{2}\times 2\sqrt{t} + 7\int \frac{1}{\sqrt{(x+2)^2+6}}\,dx$$

$$= 5\sqrt{x^2+4x+10} + 7\log\left|x+2+\sqrt{x^2+4x+10}\right| + C$$

4. $\int_0^{\pi} \frac{x\tan x}{\sec x+\tan x}\,dx$

Solution

$$I = \int_0^{\pi} \frac{x\tan x}{\sec x+\tan x}\,dx \quad \text{-------------------(1)}$$

Again I = $\int_0^{\pi} \frac{(\pi-x)\tan(\pi-x)}{\sec(\pi-x)+\tan(\pi-x)}\,dx$ Using the property $\int_0^{a} f(x)dx = \int_0^{a} f(a-x)dx$

$$I = \int_0^{\pi} \frac{(\pi-x)\tan x}{\sec x+\tan x}\,dx \quad \text{-------------------- (2)}$$

Adding (1) and (2) we get

$$2I = \int_0^{\pi} \frac{x \tan x}{\sec x+\tan x}\, dx + \int_0^{\pi} \frac{(\pi-x)\tan x}{\sec x+\tan x}\, dx$$

$$= \pi \int_0^{\pi} \frac{\tan x}{\sec x+\tan x} dx = \pi \int_0^{\pi} \frac{\tan x\,(\sec x-\tan x)}{\sec^2 x-\tan^2 x}\, dx$$

$$= \pi \int_0^{\pi} (\tan x \sec x - \tan^2 x)\, dx = \pi \int_0^{\pi} (\tan x \sec x - \sec^2 x + 1) dx$$

$$= \pi[\sec x - \tan x + x]_0^{\pi}$$

$$I = \pi\left(\frac{\pi}{2} - 1\right)$$

5. $\int \sqrt{tanx}$ dx **[HOTS]**

Solution

Put tan x = t^2 then sec^{2}x dx = 2t dt => dx = $\frac{2t\,dt}{1+t^4}$

$$\int \sqrt{tanx}\, dx = \int t\, \frac{2t\,dt}{1+t^4} = \int \frac{2t^2}{1+t^4}\, dt$$

$$= \int \frac{2}{\frac{1}{t^2}+t^2}\, dt \quad \text{(by dividing nr and dr by } t^2\text{)}$$

$$= \int \frac{\left(1+\frac{1}{t^2}\right)+\left(1-\frac{1}{t^2}\right)}{t^2+\frac{1}{t^2}}\, dt$$

$$= \int \frac{1+\frac{1}{t^2}}{t^2+\frac{1}{t^2}}\, dt + \int \frac{1-\frac{1}{t^2}}{t^2+\frac{1}{t^2}}\, dt$$

$$= \int \frac{1+\frac{1}{t^2}}{\left(t-\frac{1}{t}\right)^2+2}\, dt + \int \frac{1-\frac{1}{t^2}}{\left(t+\frac{1}{t}\right)^2-2}\, dt$$

$$= \int \frac{du}{u^2+2} + \int \frac{dv}{v^2-2}$$

{ 1st integral put $t-\frac{1}{t}$ = u, then $\left(1+\frac{1}{t^2}\right) dt = du$,

2nd integral put $t+\frac{1}{t} = v$ then $\left(1-\frac{1}{t^2}\right) dt = dv$ }

$$= \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{u}{\sqrt{2}}\right) + \frac{1}{2\sqrt{2}} log \left|\frac{v-\sqrt{2}}{v+\sqrt{2}}\right| + C$$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{t-\frac{1}{t}}{\sqrt{2}}\right) + \frac{1}{2\sqrt{2}} log \left|\frac{t+\frac{1}{t}-\sqrt{2}}{t+\frac{1}{t}+\sqrt{2}}\right| + C$$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{t^2-1}{\sqrt{2}\,t}\right) + \frac{1}{2\sqrt{2}} log \left|\frac{t^2+1-\sqrt{2}\,t}{t^2+1+\sqrt{2}\,t}\right| + C$$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{tanx-1}{\sqrt{2\,tanx}}\right) + \frac{1}{2\sqrt{2}} log \left|\frac{tanx+1-\sqrt{2tanx}}{tanx+1-\sqrt{2tanx}}\right| + C$$

PRACTICE PROBLEMS

1 Mark Questions

1. Evaluate : $\int(\sin^2 x - \cos^2 x)\,dx$ **Ans:** $-\frac{1}{2}\sin 2x + c$

2. Evaluate : $\int(2x - 3\cos x + e^x)\,dx$ **Ans:** $x^2 - 3\sin x + e^x + c$

3. Evaluate : $\int_1^{\infty}\frac{1}{x^2+1}dx$ **Ans:** $\frac{\pi}{4}$

4. Evaluate: $\int\frac{\cos 2x+2\sin^2 x}{\cos^2 x}dx.$ **Ans:** $\tan x + c$

5. The value of $\int e^x.\sec x\,(1+\tan x)dx$ is

(a) $e^x\cos x + c$ (b) $e^x\sec x + c$ (c) $e^x\sin x + c$ (d) $e^x\tan x + c$

6. The value of $\int\frac{\sin^6 x}{\cos^8 x}dx.$ is

(a) $\tan^7 x + c$ (b) $\frac{\tan^7 x}{7} + c$ (c) $\frac{\tan 7x}{7} + c$ (d) $\sec^2 x + c$

7. The value of $\int e^x\sin x(\sin x + 2\cos x)\,dx$. Is

(a) $e^x\sin^2 x + c$ (b) $e^x\sin x + c$ (c) $e^x\sin 2x + c$ (d) None of these.

8. The value of $\int\frac{e^x(1+x)}{\cos^2(e^x x)}dx$

(a) $-\cot(e^x x) + c$ (b) $\tan(e^x x) + c$ (c) $\tan(e^x) + c$ (d) $\cot(e^x) + c$

9. The value of $\int_0^1 e^{2\log x}\,dx$ is

(a) 0 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

10. The value of $\int\tan^{-1}\left(\frac{\sin 2x}{1+\cos 2x}\right)dx.$ is

(a) $\frac{x^2}{2}$+c (b) $\frac{x^3}{3}$+c (c) $\frac{\tan 7x}{7} + c$ (d) $\sec^2 x + c$

11. If $\int_0^1(3x^2 + 2x - 2k)\,dx = 0$, then the value of k. is

(a) 1 **(b)** 2 **(c)** 3 **(d)** 4

12. The value of $\int_1^{\sqrt{3}}\frac{1}{1+x^2}dx$ is ……………………

13. The value of $\int\frac{1}{e^x+e^{-x}}\,dx$ is ……………………

14. The value of $\int_0^{\frac{\pi}{2}}\cos^2 x\,dx$ is ……………………

15. The value of $\int_0^{\frac{\pi}{2}}\frac{2^{\sin x}}{2^{\sin x}+2^{\cos x}}$ is ……………………

16. The value of $\int_0^{\frac{\pi}{2}}\frac{\sin x}{\sin x+\cos x}dx$ is ……………………

17. The value of $\int(\sin^2 x - \cos^2 x)\,dx$ is ……………………

18. If $f(x)$ is an odd function, then $\int_{-a}^{a}f(x)dx$ is ………..

19. If $\int_0^{\pi}x.f(sinx)dx = \text{A}\int_0^{\pi}f(\sin x)$, then find the value of A.

20. Evaluate : $\int(3x-4)^3dx$ **Ans:** $\frac{1}{12}(3x-4)^4 + c$

21. Evaluate: $\int_{-2}^{1} \frac{|x|}{x} dx$ **Ans:** $\boldsymbol{-1}$

22. Evaluate: $\int_{2}^{3} \frac{dx}{x^2-1}$ **Ans:** $\boldsymbol{\frac{1}{2} log \frac{3}{2}}$

23. Evaluate: $\int_{0}^{\frac{\pi}{4}} \sin 2x\, dx$ **Ans:** $\boldsymbol{\frac{1}{2}}$

24. Evaluate: $\int \operatorname{cosec} x\,(\cot x - 1)e^x dx$ **Ans :** $\boldsymbol{-e^x cosec\, x + c}$

25. Evaluate: $\int 5^x\, dx$ **Ans:** $\boldsymbol{\frac{5^x}{log\, 5} + c}$

26. If $\mathrm{f}(x) = \int_{0}^{x} t \sin t\, dt$, then write the value of $\mathrm{f}'(x)$ **Ans:** $\boldsymbol{x\, sin\, x}$

27. Evaluate : $\int_{0}^{\frac{\pi}{2}} \sin^2 x dx$ **Ans:** $\boldsymbol{\frac{\pi}{4}}$

28. Evaluate: $\int \frac{1}{x+x\ \log x} dx$ **Ans:** $\boldsymbol{log(1 + logx) + c}$

2 Marks Questions

29. Evaluate: $\int \frac{10x^9+10^x log_e 10}{10^x+x^{10}} dx$ **Ans:** $\boldsymbol{log}|10^x + x^{10}| + \boldsymbol{c}$

30. Evaluate: $\int \frac{e^x(1+x)}{\cos^2(e^x x)} dx$ **Ans:** $\boldsymbol{tan}(e^x x) + \boldsymbol{c}$

31. Evaluate: $\int \frac{dx}{x \cos^2(1+\log x)}$ **Ans:** $\boldsymbol{tan}(1 + \log x) + \boldsymbol{c}$

32. Evaluate: $\int \frac{\sec^2(\log x)}{x} dx.$ **Ans :** $\boldsymbol{tan}(\mathbf{log}\, \boldsymbol{x}) + \boldsymbol{c}$

33. Find : $\int \frac{\sin^2 x - \cos^2 x}{\sin x \cos x} \mathrm{d}x$. **Ans:** $\mathbf{log\ sec}\, \boldsymbol{x}.\mathbf{cosec}\, \boldsymbol{x} + \boldsymbol{c}$

34. Find: $\int_{-\frac{\pi}{4}}^{0} \frac{1+\tan x}{1-\tan x} dx.$ **Ans:** $\boldsymbol{\frac{1}{2} log\, 2}$

35. Find : $\int \frac{dx}{5-8x-x^2}$ **Ans:** $\boldsymbol{\frac{1}{2\sqrt{21}} log \left|\frac{\sqrt{21}+4+x}{\sqrt{21}-4-x}\right| + c}$

36. Find : $\int x \,.\tan^{-1} x\, dx.$ **Ans:** $\frac{x^2}{2}\tan^{-1}x - x + \frac{1}{2}\tan^{-1}x + C$

37. Evaluate: $\int e^{2x}.\sin(3x+1)dx$ **Ans:** $\frac{2\, e^{2x}.\sin(3x+1)}{13} - \frac{3\, e^{2x}.\cos(3x+1)}{13} +$ **C**

38. Evaluate: $\int x\sqrt{x+2}\ dx$ **Ans:** $\boldsymbol{\frac{2}{5}(x+2)^{\frac{5}{2}} - \frac{4}{3}(x+2)^{\frac{3}{2}}} + \mathbf{c}$

39. Evaluate: $\int \tan^2(2x-3)dx$ **Ans:** $\boldsymbol{x - \frac{1}{2} tan(2x-3) + c}$

40. Evaluate : $\int_{2}^{4} \frac{x}{x^2+1}\ dx$ **Ans:** $\boldsymbol{\frac{1}{2} log \frac{17}{5}}$

41. Evaluate: $\int_{e}^{e^2} \frac{dx}{x \log x}$ **Ans:** $\boldsymbol{log\, 2}$

42. If $\int_{0}^{a} \frac{1}{4+x^2}\ dx = \frac{\pi}{8}$, find the value of a. **Ans: 2**

43. Evaluate: $\int \frac{(x-4)}{(x-2)^3} e^x\, dx$ **Ans:** $\boldsymbol{e^x \frac{1}{(x-2)^2} + c}$

44. Evaluate: $\int \boldsymbol{\frac{1+\sin 2x}{1+\cos 2x} e^{2x} dx}$ **Ans:** $\boldsymbol{\frac{1}{2} e^{2x} tan\, x + c}$

45. Evaluate: $\int_{1}^{2} \left(\frac{1}{x} - \frac{1}{2\,x^2}\right) e^{2x} dx$ **Ans:** $\boldsymbol{\frac{1}{4} e^2(e^2 - 2)}$

46. Evaluate: $\int \boldsymbol{\frac{dx}{x(x^3+8)}}$ **Ans:** $\boldsymbol{\frac{1}{24} log \left|\frac{x^3}{x^3+8}\right| + c}$

3 Marks Questions

47. Find : $\int \frac{x\,dx}{1+x\tan x}$ **Ans:** $\log|\boldsymbol{\cos x + x\sin x}| + c$

48. Find: $\int \frac{x^4}{(x-1)(x^2+1)}dx$ **Ans:** $\frac{x^2}{2} + x + \frac{1}{2}\log|x-1| - \frac{1}{4}\log|x^2+1| - \frac{1}{2}\tan^{-1}x + c$

49. Evaluate : $\int_0^{\frac{\pi}{2}} \frac{5\sin x + 3\cos x}{\sin x + \cos x}dx$ **Ans:** 2π

50. Find: $\int \left[\log(\log x) + \frac{1}{(\log x)^2}\right] dx$ **Ans:** $x\log(\log x) - \frac{x}{\log x} + c$

51. Evaluate : $\int_0^{\pi} \frac{4x\sin x}{1+\cos^2 x}\,dx$ **Ans:** π^2

52. Evaluate: $\int_{-1}^{1} |x\cos\pi x|\,dx$ **Ans:** $\frac{2}{\pi}$

53. Evaluate: $\int_{\frac{1}{3}}^{1} \frac{(x-x^3)^{\frac{1}{3}}}{x^4}dx$ **Ans:** 6

54. Evaluate: $\int \frac{3x+1}{\sqrt{5-2x-x^2}}dx$ **Ans:** $-3\sqrt{5-2x-x^2} - 2\sin^{-1}\left(\frac{x+1}{\sqrt{6}}\right) + c$

55. Evaluate : $\int \frac{6x+7}{\sqrt{(x-5)(x-4)}}dx$ **Ans:** $6\sqrt{x^2-9x+20} + 34\,l0g\left|x - \frac{9}{2} + \sqrt{x^2-9x+20}\right| + c$

56. Evaluate $\int \frac{1}{1-\tan x}dx$ **Ans:** $\frac{x}{2} - \frac{1}{2}\log|\cos x - \sin x| + c$

57. Evaluate $\int \frac{1}{1+\cot x}dx$ **Ans:** $\frac{x}{2} - \frac{1}{2}\log|\cos x + \sin x| + c$

58. Evaluate : $\int \frac{x\cos^{-1}x}{\sqrt{1-x^2}}dx$ **Ans:** $-\left[\sqrt{1-x^2}\cos^{-1}x + x\right] + c$

59. Evaluate : $\int \frac{\log x}{(1+\log x)^2}dx$ **Ans:** $\frac{x}{(\log x+1)} + c$

60. Evaluate : $\int \frac{dx}{x^3+x^2+x+1}$ **Ans:** $\frac{1}{2}\log|x+1| - \frac{1}{4}\log|x^2+1| + \frac{1}{2}\tan^{-1}x + c$

61. Find $\int \frac{x^2+1}{x^4+1}dx$ **Ans:** $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}\,x}\right) + c$

62. Evaluate: $\int \frac{2}{(1-x)(1+x^2)}dx$ **Ans:** $\frac{1}{2}\log|1+x^2| - \log|1-x| + \tan^{-1}x + c$

63. Find: $\int \frac{1-\cos x}{\cos x\,(1+\cos x)}dx$ **Ans:** $\log|\sec x + \tan x| - 2\tan\frac{x}{2} + c$

64. Find: $\int \frac{\cos\theta}{(4+\sin^2\theta)(5-4\cos^2\theta)}d\theta$ **Ans:** $-\frac{1}{30}\tan^{-1}\left(\frac{\sin\theta}{2}\right) + \frac{2}{15}\tan^{-1}(2\sin\theta) + c$

65. Find: $\int \frac{2\cos x}{(1-\sin x)(1+\sin^2 x)}dx$ **Ans:** $-\log(1-\sin x) + \frac{1}{2}\log(1+\sin^2 x) + \tan^{-1}(\sin x) + c$

66. Evaluate : $\int \frac{e^x\,dx}{(e^x-1)^2(e^x+2)}$ **Ans:** $\frac{1}{9}\log\left|\frac{(e^x+2)}{(e^x-1)}\right| - \frac{1}{3(e^x-1)} + c$

67. Evaluate: $\int_0^{\pi} \frac{x\tan x}{\sec x\,.\mathrm{cosec}\,x}dx$ **Ans:** $\frac{\pi^2}{4}$

68. Find : $\int \frac{x^3-1}{x^3+x}dx$ **Ans:** $x - \log|x| - \tan^{-1}x + \frac{1}{2}\log(x^2+1) + c$

69. Evaluate: $\int_0^{1} \frac{\log(1+x)}{1+x^2}dx$ **Ans:** $\frac{\pi}{8}\log 2$

70. Evaluate : $\int_0^{\pi} \frac{x\,dx}{a^2\cos^2 x + b^2\sin^2 x}$ **Ans:** $\frac{\pi^2}{2ab}$

71. Evaluate: $\int_0^{\frac{\pi}{2}} (2\log\sin x - \log\sin 2x)\,dx$ **Ans:** $-\frac{\pi}{2}\log 2$

72. Find: $\int \frac{1}{\sin x \cos^3 x} \mathrm{d}x$ **Ans:** $\mathbf{\log|\tan x| + \frac{1}{2}\tan^2 x + c}$

73. Prove that: $\int_0^1 \log\left(\frac{1}{x} - 1\right) dx = 0$

74. Evaluate $\int_0^{\frac{\pi}{2}} \frac{1}{1+\sqrt{\cot x}} dx$ **Ans:** $\boldsymbol{\frac{\pi}{4}}$

75. Evaluate : $\int \frac{\sqrt{x^2+1}[\log(x^2+1)-2\log x]}{x^4} dx$ **Ans:** $\boldsymbol{-\frac{1}{3}\left(1+\frac{1}{x^2}\right)^{\frac{3}{2}}\left[\log\left(1+\frac{1}{x^2}\right)\right]-\frac{2}{3}+c}$

76. Evaluate: $\int_1^4 [|x-1| + |x-2| + |x-3|]\, dx$ **Ans:** $\boldsymbol{\frac{19}{2}}$

77. Evaluate: $\int_0^{\frac{\pi}{4}} \log(1+\tan x)\, dx$ **Ans:** $\boldsymbol{\frac{\pi}{8}\log 2}$

78. Evaluate: $\int_0^{\pi} \frac{1}{5+4\cos x} dx$ **Ans:** $\boldsymbol{\frac{\pi}{3}}$

79. Evaluate: $\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x} dx$ **Ans:** $\boldsymbol{\frac{\pi^2}{4}}$

80. Evaluate: $\int_0^{\pi} \frac{x}{1+\sin x}$ **Ans:** $\boldsymbol{\pi}$

81. Evaluate: $\int \frac{1}{\cos(x-a)\cos(x-b)} dx$ **Ans:** $\boldsymbol{\frac{1}{\sin(a-b)} \log\left|\frac{\cos(x-a)}{\cos(x-b)}\right| + c}$

82. Integrate the function $\frac{\cos(x+a)}{\sin(x+\mathrm{b})}$ w.r.t. x. **Ans**: $\boldsymbol{\cos(a-b)\log|\sin(x+b)| - x\sin(a-b) + C}$

83. Evaluate: $\int \frac{1}{\sin^2 x + \sin 2x} dx$ **Ans:** $\boldsymbol{\frac{1}{2}\log\left|\frac{\tan x}{\tan x + 2}\right| + c}$

84. Evaluate $\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9+16\sin 2x} dx$ **Ans:** $\boldsymbol{\frac{1}{40}\log 9}$

85. Evaluate: $\int_0^{\frac{\pi}{2}} \left(\sqrt{\tan x} + \sqrt{\cot x}\right) dx$ **Ans :** $\boldsymbol{\sqrt{2}\pi}$

86. Prove that : $\int_0^{\frac{\pi}{2}} \sin 2x \log\tan x = 0$

87. Prove that : $\int_{-a}^{a} \sqrt{\frac{a-x}{a+x}}\, dx = a\pi$

88. Evaluate: $\int \tan^{-1}\sqrt{\frac{1-x}{1+x}} dx$ **Ans :** $\boldsymbol{\frac{1}{2}\left(x\cos^{-1}x - \sqrt{1-x^2}\right) + c}$

89. Evaluate the following indefinite integral: $\int \frac{\sin\phi}{\sqrt{\sin^2\phi + 2\cos\phi + 3}} \mathrm{d}\phi$ **Ans:** $\boldsymbol{-\sin^{-1}\frac{\cos\phi - 1}{\sqrt{5}} + c}$

90. Evaluate the following definite integral: $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x} dx$ **Ans:** $\boldsymbol{\pi^2}$

91. Find: $\int \frac{dx}{x^3(x^5+1)^{\frac{3}{5}}}$ **Ans:** $\boldsymbol{-\frac{1}{2}\left(1+\frac{1}{x^5}\right)^{\frac{2}{5}} + c}$

92. Evaluate: $\int_0^{\frac{\pi}{2}} \frac{\cos^2 x\, dx}{1+3\sin^2 x}$ **Ans:** $\boldsymbol{\frac{\pi}{6}}$

93. Evaluate: $\int_0^1 \cot^{-1}(1-x+x^2)\, dx$ **Ans:** $\boldsymbol{-\frac{\pi}{2}\log 2}$

94. Find: $\int_0^{\frac{1}{\sqrt{2}}} \frac{\sin^{-1}x}{(1-x^2)^{\frac{3}{2}}} \mathrm{d}x$ **Ans:** $\boldsymbol{\frac{\pi}{4} - \frac{1}{2}\log 2}$

95. Evaluate: $\int_0^{\pi} e^{2x}.\sin\left(\frac{\pi}{4}+x\right) dx$ **Ans:** $\boldsymbol{-\frac{e^{2\pi}+1}{5\sqrt{2}}}$

96. Find $\int \frac{\sqrt{x}}{\sqrt{a^3-x^3}} dx$ **Ans:** $\boldsymbol{\frac{2}{3}\sin^{-1}\sqrt{\frac{x^3}{a^3}} + c}$

97. Evaluate: $\int_0^{\pi} \frac{x \sin x}{1+3\cos^2 x}\, dx$ **Ans:** $\frac{\sqrt{3}\pi^2}{9}$

98. Evaluate : $\int_0^{\pi} \frac{x}{1+\sin\alpha . \sin x} dx$ **Ans:** $\frac{\pi(\pi-2\alpha)}{2\cos\alpha}$

5 Marks Questions

99. Evaluate: $\int \frac{\sin^6 x+\cos^6 x}{\sin^2 x .\ \cos^2 x}\, dx$ **Ans:** $\boldsymbol{\tan x - \cot x - 3x + c}$

100. Evaluate: $\int \frac{x^2}{x^4+x^2-2}\, dx$ **Ans:** $\frac{2}{3\sqrt{2}} \tan^{-1}\frac{x}{\sqrt{2}} + \frac{1}{3} \log\left|\frac{x-1}{x+1}\right| + c$

101. Evaluate: $\int_0^{\frac{\pi}{2}} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx,$ **Ans:** $\frac{\pi^2}{16}$

102. Evaluate: $\int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin x + \cos x} dx$ **Ans:** $\frac{1}{\sqrt{2}} \log(\sqrt{2}+1)$

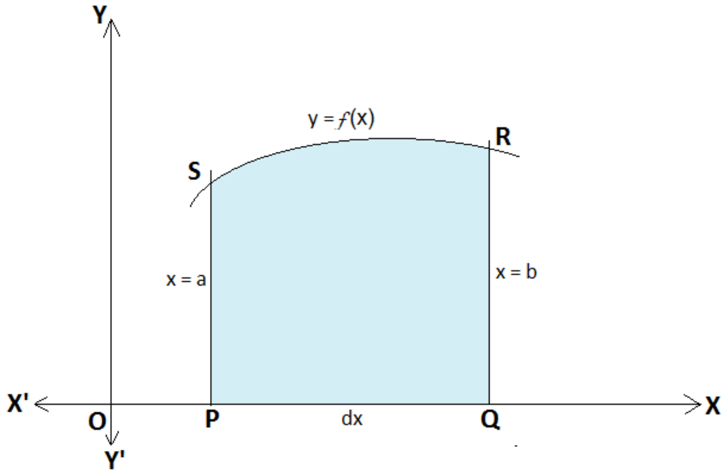
103. Find : $\int_0^{\frac{\pi}{4}} \frac{dx}{\cos^3 x \sqrt{2\sin 2x}}$ **Ans:** $\frac{6}{5}$

104. Evaluate : $\int_0^{\frac{\pi}{2}} \log \sin x\, dx$ **Ans:** $-\frac{\pi}{2} \log 2$

APPLICATION OF INTEGRATION

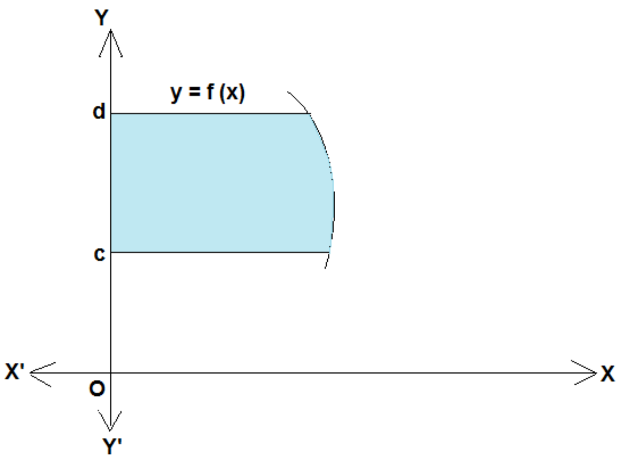
INTRODUCTION

Area under Simple Curves: (i)



Area bounded by the curve y = f(x), the x-axis and between the ordinates at x = a and x = b is given by Area = $\int_a^b y\,dx = \int_a^b f(x)\,dx$

(ii)



Area bounded by the curve y = f(x), the y axis and between abscissas at y = c and y = d is given by **Area =** $\int_c^d x\,dx = \int_c^d g(y)dy$

Where **y =** $f(x) \Rightarrow x = g(y)$

Note: If area lies below x-axis or to left side of y-axis, then it is negative and in such a case we like its absolute value. (Numerical value)

<u>Finding the area enclosed between a curve, X- axis and two ordinates or a curve , Y- axis and two abscissa</u>

WORKING RULE

1. Draw the rough sketch of the given curve

2. Find whether the required area is included between two ordinate or two abscissa

3. (a) If the required area is included between two ordinates x = a and x= b then use

the formula $\int_a^b y\,dx$

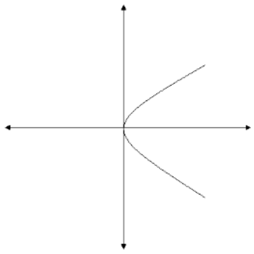
(b) If the required area is included between two abscissa y = c and = d then use the

formula $\int_c^d x\,dy$

<u>SOME IMPORTANT POINTS TO BE KEPT IN MIND FOR SKETCHING THE GRAPH</u>

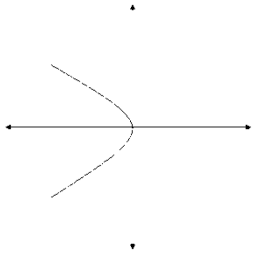
1. y^2 = 4ax is a parabola with vertex at origin,

symmetric to X axis and right of origin



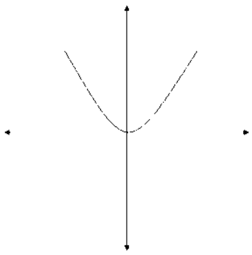
2. y^2 = - 4ax is a parabola with vertex at origin,

symmetric to X axis and left of origin



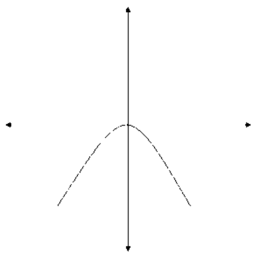
3. x^2 = 4ay is a parabola with vertex at origin,

symmetric to y axis and above origin



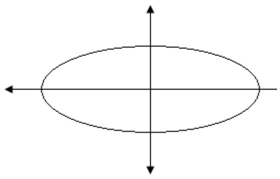
4. x^2 = - 4ay is a parabola with vertex at origin,

symmetric to y axis and below origin



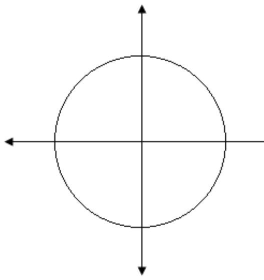
5. $\frac{x^2}{a^2}+\frac{y^2}{b^2}$ = 1 is an ellipse symmetric to both axis,

Cut x axis at ($\pm$a,0) and y axis at (0,$\pm b$)



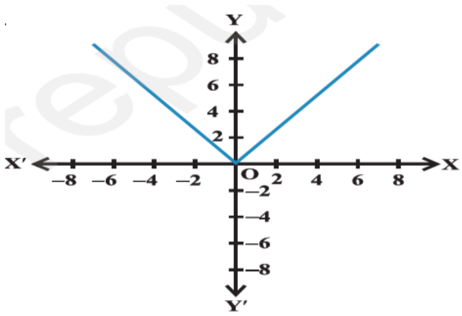
6. $x^2 + y^2 = r^2$ is a circle symmetric to both the axes

with centre at origin and radius r



7. $(x-h)^2 + (y-k)^2 = r^2$ is a circle with centre at (h,k) and radius r.

8. ax +by + c = 0 representing a straight line



$f(x) = |x|$

9. Graph of $y = |x|$

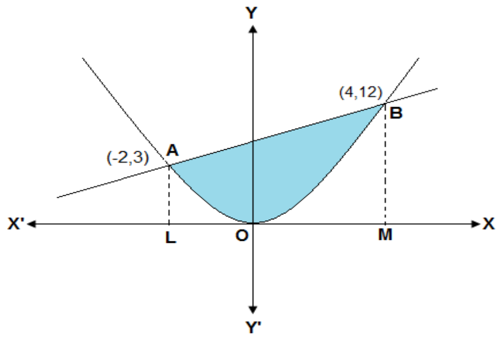
SOLVED PROBLEMS

1. Find the area cut off from the parabola $4y = 3x^2$ by the line $2y = 3x + 12$

Solution: Given $4y = 3x^2$ and $3x - 2y + 12 = 0$

Solve both the equation we get the point of intersecction of bo

(-2,3) and (4,12)



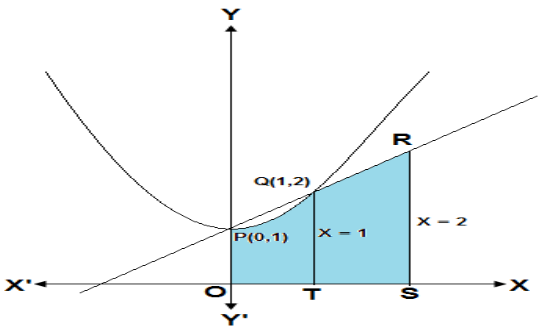
Required area = area of AOBA

$$= \int_{-2}^{4}\left[\frac{3x+12}{2} - \frac{3x^2}{4}\right] dx$$

= 27 sq.unts

2. Find the area of the region { (x,y): $0 \le y \le x^2 + 1, 0 \le y \le x+1, 0 \le x \le 2\}$

Solution: Sketch the region whose area is to be found out.



The point of intersection of $y = x^2 + 1$ and $y = x+1$ are he points (0,1) and (1,2)

The required area = area of the region OPQRSTO

= area of the region OTQPO + area of the region TSRQT

$$= \int_0^1 (x^2 + 1)\,dx + \int_1^2 (x + 1)\,dx$$

$$= \left[\frac{x^3}{3} + x\right]_0^1 + \left[\frac{x^2}{2} + x\right]_1^2$$

$$= \frac{23}{6} \text{ sq.units}$$

PRACTICE PROBLEMS

2 Marks Questions

1. Find the area of the region bounded by the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$.
2. Find the area enclosed by the circle $x^2+y^2=4$.
3. Find the area of the region bounded by $y^2=9\,x, x=2, x=4$ and the x – axis in the first quadrant.

3 Marks Questions

4. Sketch the graph of $y=|x+3|$ and hence evaluate $\int_{-6}^{0}|x+3|dx$ **Ans: 9 sq.units**
5. Find the area of the region bounded by the parabola $y=x^2$ and $y=|x|$ **Ans: $\frac{1}{3}$ sq.units**
6. Find the area of the region in the first quadrant enclosed by x – axis, line $x=\sqrt{3}\,y$ and the circle $x^2+y^2=4$. **Ans: $\frac{\pi}{3}$ sq.units**
7. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$and the line $\frac{x}{3}+\frac{y}{2}=1$ **Ans: $\frac{3}{2}(\pi-2)$sq.units**
8. Find the area of the region bounded by the curves $x^2=4y$ and the line$x=4y-2$ **Ans: $\frac{9}{8}$ sq.units**
9. Find the area of the region bounded between the parabola $4y=3x^2$ and the line $3x-2y+12=0$ **Ans: 27 sq.units**
10. Using integration, find the area of the region bounded by the line $x-y+2=0$ and the curve $x=\sqrt{y}$ and y-axis
11. If the area bounded by the parabola $\mathrm{y}^2=16\mathrm{a}x$ and the line $y=4\,\mathrm{m}\,x$ is $\frac{a^2}{12}$ sq. units, then using integration, find the value of m. **Ans: $\mathbf{m}=2\sqrt{2}$**
12. Find the area of the region $\{(x,y):\ x^2+y^2\le 4, x+y\ge 2\}$, using the method of integration. **Ans: $(\pi-2)$sq.units**
13. Using integration, find the area of the region $\{(x,y): 9x^2+y^2\le 36 \text{ and } 3x+y\ge 6\}$ **Ans: $(3\pi-6)$sq.units**

DIFFERENTIAL EQUATIONS

INTRODUCTION

1. **Problems based on the order and degree of the differential equations**

Working rule

(a) In order to find the order of differential equations, see the highest derivative in the given differential equation. Write down the order of this highest order derivatives.

(b) In order to find the degree of a differential equation write down the power of the highest order derivative after making the derivatives occurring in the given differential equation free from radicals and fractions.

2. **Problems based on solution of differential equation in which variables are separable.**

Working rule

This differential equation can be solved by the variable separable method which can be put in the form

$$\frac{dy}{dx} = f(x).g(y).$$

i.e., in which $\frac{dy}{dx}$ can be expressed as the product of two functions, one of which is a function of x only and the other a function of y only.

In order to solve the equation $\frac{dy}{dx} = f(x).g(y)$. Write down this equation in the form $\frac{dy}{g(y)} = f(x).dx$, then the solution will be $\int \frac{dy}{g(y)} = \int f(x)dx + c$, where C is an arbitrary constant.

3. **Problems based on solution of differential equations which are homogeneous.**

Working rule

(a) Write down the given differential equation in the form $\frac{dy}{dx} = f(x, y)$

(b) If $f(kx, ky) = f(x, y)$. then differential equation is homogeneous.

(c) In order to solve, put $y = vx$, so that $\frac{dy}{dx} = v + x\frac{dv}{dx}$ and then separate the variables x and v.

(d) Now solve the obtained differential equation by the variable separable method. At the end put $\frac{y}{x}$ in place of v.

4. **Working Rule for Linear Differential Equation of first degree:**

Type $\frac{dy}{dx} + Py = Q$, where P and Q are constants or function of x only

General solution is $y.IF = \int(Q.IF)dx + C$, where $IF = e^{\int Pdx}$.

Note: Particular solution can be obtained after getting the value of parameter C by substituting the given initial values of the variables.

SOLVED PROBLEMS

1. **Solve the differential equation (x + y) dy + (x – y) dx = 0**

Solution:

(x + y) dy + (x – y) dx = 0

$$\frac{dy}{dx} = \frac{y-x}{x+y}$$

Put y = vx

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\therefore \; v + x\frac{dv}{dx} = \frac{vx-x}{x+vx}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{v-1}{v+1}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{v-1}{v+1} - v$$

$$\Rightarrow x\frac{dv}{dx} = \frac{-(1+v^2)}{1+v}$$

$$\Rightarrow \frac{1+v}{1+v^2}\,dv = -\frac{dx}{x}$$

Integrating both sides we get

$$\int \frac{1+v}{1+v^2}\,dv = -\int \frac{dx}{x}$$

$$\int \frac{dv}{1+v^2} + \frac{1}{2}\int \frac{2v}{1+v^2}\,dv = -\log x + c$$

$$\tan^{-1} v + \frac{1}{2}\log|1+v^2| = -\log x + c$$

$$\tan^{-1}\frac{y}{x} + \frac{1}{2}\log\left(1+\frac{y^2}{x^2}\right) = -\log x + c$$

$$\tan^{-1}\frac{y}{x} + \frac{1}{2}\log\left(x^2+y^2\right) = c$$

2. **Solve** $x\sqrt{1-y^2}\,dx + y\sqrt{1-x^2}\,dy = 0$

Solution:

$$x\sqrt{1-y^2}\,dx = -\,y\sqrt{1-x^2}\,dy$$

$$\frac{x}{\sqrt{1-x^2}}\,dx = -\frac{y}{\sqrt{1-y^2}}\,dy$$

Integrating both sides

$\int \frac{x}{\sqrt{1-x^2}}dx$ = - $\int \frac{y}{\sqrt{1-y^2}}dy$

$\frac{-1}{2}\int \frac{dt}{\sqrt{t}}$ = $\frac{1}{2}\int \frac{du}{\sqrt{u}}$ (put t = 1 – x^2 and put u = 1 – y^2)

$-\sqrt{t} = \sqrt{u}$ + C

$$\sqrt{1-x^2} + \sqrt{1-y^2} = C$$

3. Solve the differential equation $\frac{dy}{dx} - \frac{1}{x}.y = 2x^2$

Solution:

The diff. eqn is in the form $\frac{dy}{dx} + Py = Q$

Where P = $\frac{-1}{x}$ and Q = $2x^2$

I.F = $e^{\int P\,dx} = e^{\int \frac{-1}{x}dx} = e^{-\log x} = \frac{1}{x}$

Multiplying both sides of diff.eqn by I.F we get

$$\frac{1}{x}\frac{dy}{dx} - \frac{1}{x^2}y = 2x$$

$$\frac{d}{dx}\left(y.\frac{1}{x}\right) = 2x$$

Integrating both sides w.r.t.x we get

$$y.\frac{1}{x} = x^2 + c$$

y = x^3 + Cx

4. Solve the diff. eqn $x\frac{dy}{dx} = y - x \tan\frac{y}{x}$

Solution:

$$\frac{dy}{dx} = \frac{y - x\tan\frac{y}{x}}{x}$$

Put y = vx => $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$$\therefore \quad v + x\frac{dv}{dx} = \frac{vx - x\,tanv}{x}$$

- ⇨ $v + x\frac{dv}{dx}$ = v – tan v
- ⇨ $x\frac{dv}{dx} = -tan\,v$
- ⇨ cot v dv = $-\frac{dx}{x}$

  Integrating both sides ,we get

$\int \cot v \, dv \quad = -\int \frac{dx}{x}$

log sin v = $-$ log x + log C

log [x. sin $\left(\frac{y}{x}\right)$] $= \log c$

x sin $\left(\frac{y}{x}\right)$ = C

5. $xy \frac{dy}{dx} \; = \; (x+2)(y+2)$ **, find the equation of the curve passing through the points (1, -1)**

Solution: $xy \frac{dy}{dx} \; = \; (x+2)(y+2)$

⇨ $\frac{y}{y+2} \, dy = \frac{x+2}{x} \, dx$

⇨ $\left(1 - \frac{2}{y+2}\right) dy = \left(1 + \frac{2}{x}\right) dx$

⇨ **Integrating both sides we get**

⇨ **Y – 2 log (y + 2) = x + 2 log x + C**

⇨ **The curve is passing through the the point (1,-1)**

$-$1 – 2 log 1 = 1 + 2 log 1 + C => C = $-$2

The equation of the line is $y - x = 2 \log$[x(y+2)] – 2

6. **Solve the differential equation** $\left(x^2 - 1\right)\frac{dy}{dx} + 2xy \; = \; \frac{2}{x^2-1}$

Solution

$$\left(x^2 - 1\right)\frac{dy}{dx} + 2xy \; = \; \frac{2}{x^2 - 1}$$

$\frac{dy}{dx} + \left(\frac{2x}{x^2-1}\right).y = \frac{2}{(x^2-1)^2}$

I.F. $= e^{\int \frac{2x}{x^2-1}dx} \quad = \quad e^{\log(x^2-1)} = \left(x^2 - 1\right)$

⇨ $\frac{d}{dx}\left(y\left(x^2 - 1\right)\right) \; = \; \frac{2}{x^2-1}$

Integrating both sides w.r.t x we get

$y\left(x^2 - 1\right) \; = \; \int \frac{2}{x^2-1} \, dx$

$y\left(x^2 - 1\right) = \log\left(\frac{x-1}{x+1}\right) + C$

PRACTICE PROBLEMS

1 Mark Questions

1. If $\sin x$ is an integrating factor of differential equation $\frac{dy}{dx} + \mathrm{Py} = \mathrm{Q}$, then P is…………

2. Find the integrating factor of the following differential equation :
$x \log x \frac{dy}{dx} + y = 2 \log x$ **Ans: log$x$**

3. If m and n are the order and degree, respectively of the differential equation $y\left(\frac{dy}{dx}\right)^3 + x^3\left(\frac{d^2y}{dx^2}\right)^2 - xy = \sin x$, then write the value of m + n **Ans:4**

4. Find the sum of the degree and the order for the following differential equation :
$\frac{d}{dx}\left[\left(\frac{d^2y}{dx^2}\right)^4\right] = 0$ **Ans: order 3, degree 1**

5. Find the product of the order and degree of the following differential equation: $x\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 + y^2 = 0$ **Ans: 4**

6. Determine order and degree of differential equation: $\frac{d^4y}{dx^4} + \sin(y''') = 0$
Ans: order 4, degree not defined

3 Marks Questions

7. Solve the differential equation :$\frac{dy}{dx} + y = \cos x - \sin x$.**Ans:**$\boldsymbol{y = \cos x + Ce^{-x}}$

8. Solve that the differential equation**:** $x\frac{dy}{dx} + y - x + xy \cot x = 0$; $x \neq 0$
Ans: $\boldsymbol{xysinx = -xcosx + sinx + c}$

9. **Find particular solution of the differential equation:** $\boldsymbol{\frac{dy}{dx} = 1 + x + y + xy}$ **,given that when** $\boldsymbol{y(1) = 0}$ **Ans**: $\boldsymbol{log|1 + y| = x + \frac{x^2}{2} - \frac{3}{2}}$

10. Solve the differential equation: $\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$ given that $y = 1$, when $x = 0$
Ans. $\boldsymbol{\tan^{-1}y = \frac{x^3}{3} + x + C, C = \frac{\pi}{4}}$

11. Find the particular solution of the differential equation: $\frac{dy}{dx} = \frac{xy}{x^2+y^2}$ given that $y = 1$ when $x = 0$ **Ans:** $\boldsymbol{logy = -\frac{x^2}{2y^2}}$

12. Solve the differential equation:$(1+x^2)\frac{dy}{dx} + y = e^{tan^{-1}x}$ **Ans:**$\boldsymbol{y = \frac{1}{2}e^{tan^{-1}x} + ce^{-tan^{-1}x}}$

13. Find the particular solution of the differential equation:$(x - y)\frac{dy}{dx} = (x + 2y)$, given that $y = 0$ when $x = 1$ **Ans:** $\boldsymbol{log\left|x^2 + xy + y^2\right| = 2\sqrt{3}tan^{-1}\left(\frac{2y+x}{\sqrt{3}x}\right) - \frac{1}{\sqrt{3}}\pi}$

14. Solve the following differential equation **:** $(\tan^{-1}x - y)\, dx = (1 + x^2)\, dy$
Ans:$\boldsymbol{y = (tan^{-1}x - 1) + c.e^{-tan^{-1}x}}$

15. Find the particular solution of the differential equation :
$\log\left(\frac{dy}{dx}\right) = 3x + 4y$,given that $y = 0$ when$x = 0$. **Ans:** $\boldsymbol{4e^{3x} + 3e^{-4y} - 7 = 0}$

16. Find the particular solution of the differential equation : **Ans:** $\boldsymbol{y = \sqrt{2x^2 + 1}}$
$x(1 + y^2)\, dx - y(1 + x^2)dy = 0$, given that $y = 1$ when$x = 0$.

17. Solve the differential equation :$x\frac{dy}{dx} = y - x\tan\left(\frac{y}{x}\right)$ **Ans:** $\boldsymbol{xsin\frac{y}{x} = c}$

18. Find the particular solution of differential equation $\frac{dy}{dx} + y \cot x = 2x + x^2 \cot x$, given that $y = 0$ when $x = \frac{\pi}{2}$ **Ans:** $\boldsymbol{ysinx - x^2sinx + \frac{\pi^2}{4} = 0}$

19. Solve the differential equation: $\frac{dy}{dx} - \frac{y}{x} + \operatorname{cosec}\frac{y}{x} = 0$ given that $y = 0$,when $x = 1$
Ans: $\boldsymbol{\cos\frac{y}{x} - \log x = 1}$

20. Find the particular solution of the differential equation $e^x \tan y\, dx + (2 - e^x)\sec^2 y\, dy = 0$, given that $y = \frac{\pi}{4}$ when $x = 0$. **Ans:** $\boldsymbol{y = 2 - e^x}$

21. Solve the differential equation: $\frac{dy}{dx} + 2y\tan x = \sin x$, given that $y = 0$, when $x = \frac{\pi}{3}$,
Ans: $\boldsymbol{y = \cos x - 2\cos^2 x}$

22. Find the general solution of the equation: $3e^x \tan y\, dx + (1 - e^x)\sec^2 y\, dy = 0$ **Ans:** $\boldsymbol{(e^x - 1)^3 = C\tan y}$

23. Show that the differential equation $2xy\frac{dy}{dx} = x^2 + 3y^2$ is homogeneous and solve it. **Ans:** $\boldsymbol{x^2 + y^2 = cx^3}$

24. Find the particular solution of the differential equation: $\frac{dy}{dx} + y\tan x = 3x^2 + x^3\tan x, x \neq \frac{\pi}{2}$, given that $y = 0$ when $x = \frac{\pi}{3}$ **Ans:** $\boldsymbol{y = x^3 - \frac{2\pi^3}{27}\cos x}$

25. Find particular solution of $(x + 1)\frac{dy}{dx} = 2e^{-y} - 1$, given that $y = 0, x = 0$
Ans: $\boldsymbol{(x + 1)(2 - e^y) = 1}$

26. Find the general solution of the differential equation $\frac{dy}{dx} - y = \sin x$
Ans: $\boldsymbol{y = -\frac{1}{2}(\sin x + \cos x) + ce^x}$

27. Solve the following differential equation, given that $\boldsymbol{y = 0}$, when $\boldsymbol{x = \frac{\pi}{4}}$: $\boldsymbol{\sin 2x\frac{dy}{dx} - y = \tan x}$
Ans: $\boldsymbol{y = \tan x - \sqrt{\tan x}}$

28. Solve: $\frac{dy}{dx} + y\sec x = \tan x\ \left(0 \le x < \frac{\pi}{2}\right)$ **Ans:** $\boldsymbol{y(\sec x + \tan x) = \sec x + \tan x - x + c}$

29. Solve : $\boldsymbol{\frac{dy}{dx} + y\cot x = 4x\,\mathrm{cosec}\, x}$ **, given** $\boldsymbol{y = 0}$ **when** $\boldsymbol{x = \frac{\pi}{2}}$ **Ans:** $\boldsymbol{y\sin x = 2x^2 - \frac{\pi^2}{2}}$

30. Solve the following differential equation: $\left(\sqrt{1 + x^2 + y^2 + x^2y^2}\right)dx + xy\, dy = 0$.
Ans: $\boldsymbol{\sqrt{1 + y^2} + \sqrt{1 + x^2} + \frac{1}{2}\log\left|\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right| + c}$

31. Find the particular solution of the differential equation: $(1 + x^2)\frac{dy}{dx} = \left(e^{m\tan^{-1}x} - y\right)$ given that $y = 1$, when $x = 0$ **Ans:** $\boldsymbol{y.e^{\tan^{-1}x} = \frac{e^{(m+1)\tan^{-1}x}}{m+1} + \frac{m}{m+1}}$

32. Solve the differential equation: $\boldsymbol{\frac{dy}{dx}} - 3y\cot x = \sin 2x$, given $y = 2$ when $x = \frac{\pi}{2}$
Ans: $\boldsymbol{y = -2\sin^2 x + 4\sin^3 x}$

5 Marks Questions

33. Find the particular solution of the differential equation: $\frac{dy}{dx} = -\frac{x + y\cos x}{1+\sin x}$ given that $y = 1$ when $x = 0$ **Ans:** $\boldsymbol{y = -\frac{1}{2}x^2\frac{1}{(1+\sin x)} + \frac{1}{(1+\sin x)}}$

34. Solve the following differential equation: $\left[y - x\cos\left(\frac{y}{x}\right)\right]dy + \left[y\cos\left(\frac{y}{x}\right) - 2x\sin\left(\frac{y}{x}\right)\right]dx = 0$
Ans: $\boldsymbol{y^2 - 2x^2}\cos\left(\frac{y}{x}\right) = C$

35. Solve the differential equation $\left\{x\cos\left(\frac{y}{x}\right) + y\sin\frac{y}{x}\right\}ydx = \left\{y\sin\frac{y}{x} - x\cos\frac{y}{x}\right\}xdy$
Ans: $\boldsymbol{xy\cos\frac{y}{x} = A}$

36. Find the particular solution of the differential equation:
$xe^{\frac{y}{x}} - y\sin\left(\frac{y}{x}\right) + x\frac{dy}{dx}\sin\left(\frac{y}{x}\right) = 0$, given that $y = 0$, when $x = 1$ **Ans:** $\boldsymbol{\left[\sin\frac{y}{x} + \cos\frac{y}{x}\right]e^{\frac{-y}{x}}\log x^2 + 1}$

37. Solve the following differential equation :$\frac{dy}{dx} = \frac{y}{x} + \frac{\sqrt{x^2+y^2}}{x}, x > 0$**Ans:** $\boldsymbol{y + \sqrt{x^2 + y^2} = cx^2}$

38. $(x^2 + y^2)dy = xy\,dx$. If $y(1) = 1$ and $y(x_0) = e$, then find the value of x_0 **Ans:** $\boldsymbol{x_0 = \sqrt{3}e}$

Vector Algebra

Summary

1. Position vector of a point P(x,y,z) is given as $\overrightarrow{OP}(\vec{r}) = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and its magnitude by $|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$.
2. The position vector ($\vec{r}$) of a point R dividing a line segment joining the points P and Q whose position vectors are $\vec{a}$ and $\vec{b}$ respectively, in the ratio m : n
   (i) internally, is given by $\vec{r} = \frac{m\vec{b}+n\vec{a}}{m+n}$.
   (ii) externally, is given by $\vec{r} = \frac{m\vec{b}-n\vec{a}}{m-n}$.
   (iii) if R is the mid - point of PQ, then $\vec{r} = \frac{\vec{a}+\vec{b}}{2}$.
3. The scalar product (dot product) of two given vectors $\vec{a}$ and $\vec{b}$ having angle θ between them is defined as
   $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$.

   $\Rightarrow \cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$

   If $\vec{a}$ and $\vec{b}$ are perpendicular to each other then $\vec{a}.\vec{b} = 0$.
4. The vector (cross) product of two given vectors $\vec{a}$ and $\vec{b}$ having angle θ between them given as
   $\vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \sin\theta\, \hat{n}$,
   where $\hat{n}$ is a unit vector perpendicular to the plane containing $\vec{a}$ and $\vec{b}$ such that $\vec{a}, \vec{b}, \hat{n}$ form right handed system of co-ordinate axes.
   If $\vec{a}$ and $\vec{b}$ are parallel (collinear) to each other then $\vec{a} \times \vec{b} = \vec{0}$.
5. If $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ and $\vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$, then $\vec{a}.\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$.
6. If $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ and $\vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$, then
   $$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$
7. The projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a}.\vec{b}}{|\vec{b}|} = \vec{a}.\hat{b}$
8. **Direction cosines of a Vector** :
   The direction of a vector $\overrightarrow{OP}(\vec{r})$ is determined by the angles α, β, γ which it makes with OX, OY,OZ respectively. These angles are called the direction angles and their cosines are called the direction cosines. Direction cosines of a vector are denoted by *l, m, n.*
   $l = \cos\alpha$, $m = \cos\beta$, $n = \cos\gamma$
   Always, $l^2+m^2+n^2=1$ i.e., $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$
9. **Direction Ratios-** Set of any three numbers which are proportional to direction cosines are called direction ratios of the vector and are usually denoted by a, b, c.
   $\frac{l}{a} = \frac{m}{b} = \frac{n}{c}$.

10. Direction ratios of the vector $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ may be taken as a_1, a_2, a_3 and the Direction cosines are

$$l = \frac{a_1}{\sqrt{{a_1}^2 + {a_2}^2 + {a_3}^2}}, \quad m = \frac{a_2}{\sqrt{{a_1}^2 + {a_2}^2 + {a_3}^2}}, \quad n = \frac{a_3}{\sqrt{{a_1}^2 + {a_2}^2 + {a_3}^2}}$$

SECTION – A (1 MARK)

1. If a unit vector $\vec{a}$ makes angles $\frac{\pi}{3}$ with $\hat{\imath}$, $\frac{\pi}{4}$ with $\hat{\jmath}$ and an angle θ with $\hat{k}$, then find the value of θ.
2. Find the direction cosines of the vector joining the points $A\ (1, 2, -3)$ and $B(-1, -2,\ 1)$ directed from B to A.
3. Find the magnitude of each of the two vectors $\vec{a}$ and $\vec{b}$, having the same magnitude such that the angle between them is 60^0 and their scalar product is $\frac{9}{2}$.
4. Find the projection of $\vec{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$ on $\vec{b} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$
5. Find $|\vec{x}|$, if for a unit vector $\vec{a}$, $(\vec{x} - \vec{a}).(\vec{x} + \vec{a}) = 15$
6. Find a unit vector in the direction of $\vec{a} + \vec{b}$ where $\vec{a} = -\hat{\imath} + \hat{\jmath} + \hat{k}$ and $\vec{b} = 2\hat{\imath} + \hat{\jmath} - 3\hat{k}$.
7. If $|\vec{a}| = 13, |\vec{b}| = 5$ and $\vec{a}.\vec{b} = 60,$ then find $|\vec{a} \times \vec{b}|$
8. For what values of μ, the vectors $\vec{a} = 2\ \hat{\imath} + \mu\hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\imath} - 2\ \hat{\jmath} + 3\ \hat{k}$ are perpendicular to each other?
9. Find the area of the parallelogram, whose diagonals are $\vec{d}1 = 5\hat{\imath}$ and $\vec{d}_2 = 2\hat{\jmath}$.
10. Find the value of 'p' for which vectors $3\hat{\imath} + 2\hat{\jmath} + 9\hat{k}$ and $\hat{\imath} - 2p\hat{\jmath} + 3\hat{k}$ are parallel.
11. Write the coordinates of the point which is the reflection of the point $(\alpha,\ \beta,\ \gamma)$ in the XZ – plane.
12. Find the position vector of the point which divides the join of the points with position vectors $\vec{a} + 3\vec{b}$ and $\vec{a} - \vec{b}$ internally in the ratio $1 : 3$.
13. Find the vector in the direction of the vector $\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$ that has magnitude 9.
14. Find the direction cosines of the vector $\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$.

SECTION – B (2 MARKS)

15. Find the value of a + b, if the point $(2,\ a,\ 3)$, $(3, -5,\ b)$ and $(-1,\ 11,\ 9)$ are collinear.
16. Find a unit vector perpendicular to the plane of the triangle ABC, where the coordinates of its vertices are A $(3, -1, 2)$, B $(1, -1, -3)$ and C $(4, -3, 1)$.
17. The scalar product of the vector $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$ with a unit vector along the sum of the vectors $\vec{b} = 2\hat{\imath} + 4\hat{\jmath} - 5\hat{k}$ and $\vec{c} = \lambda\ \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ is equal to one. Find the value of λ.

18. Show that the direction cosines of avector equally inclined to the axes OX, OY and OZ are $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$.
19. If $\vec{a}+\vec{b}+\vec{c}=0$ and $|\vec{a}|=3$, $|\vec{b}|=5$ and $|\vec{c}|=7$, show that the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{3}$.
20. Find a unit vector perpendicular to each of the vectors $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$, where $\vec{a}=\hat{\imath}+\hat{\jmath}+\hat{k}$ and $\vec{b}=\hat{\imath}+2\hat{\jmath}+3\hat{k}$.
21. If $|\vec{a}\times\vec{b}|^2+|\vec{a}\,.\,\vec{b}|^2=400$ and $|\vec{a}|=5$, then write the value of $|\vec{b}|$.
22. The two adjacent sides of a parallelogram are $2\hat{\imath}-4\hat{\jmath}+5\hat{k}$ and $\hat{\imath}-2\hat{\jmath}-3\hat{k}$. Find the unit vector parallel to one of its diagonals. Also, find its area.
23. If $\vec{a}$ and $\vec{b}$ are two unit vectors inclined to x- axis at angles 45^0 and 135^0 respectively, then find the value of $|\vec{a}+\vec{b}|$.

SECTION – C (3 MARKS)

24. If $\vec{a}=\hat{\imath}+\hat{\jmath}+\hat{k}$ and $\vec{b}=\hat{\jmath}-\hat{k}$, find a vector $\vec{c}$ such that $\vec{a}\times\vec{c}=\vec{b}$ and $\vec{a}.\vec{c}=3$.
25. Show that the four points A, B, C and D with position vectors $4\hat{\imath}+5\hat{\jmath}+\hat{k}$, $-\hat{\jmath}-\hat{k}$, $3\hat{\imath}+9\hat{\jmath}+4\hat{k}$ and $4\,(-\hat{\imath}+\hat{\jmath}+\hat{k})$ respectively are coplanar.
26. Find the value of λ if the points A$(-1,4,-3)$,B$(3,\lambda,-5)$,C$(-3,8,-5)$and D$(-3,2,1)$are coplanar.
27. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three vectors such that $|\vec{a}|=1$, $|\vec{b}|=4$, $|\vec{c}|=2$ and $\vec{a}+\vec{b}+\vec{c}=0$, then find the value of μ, if $\mu=\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$
28. Let If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three vectors such that $|\vec{a}|=3$, $|\vec{b}|=4$ and $|\vec{c}|=5$ and each one of them is perpendicular to the sum of other two, find $|\vec{a}+\vec{b}+\vec{c}|$
29. Show that the points A, B, C with position vectors $2\hat{\imath}-\hat{\jmath}+\hat{k}$, $\hat{\imath}-3\hat{\jmath}-5\hat{k}$ and $3\hat{\imath}-4\hat{\jmath}-4\hat{k}$ respectively, are the vertices of a right- angled triangle. Hence find the area of the triangle.
30. The magnitude of the vector product of the vector $\hat{\imath}+\hat{\jmath}+\hat{k}$ with a unit vector along the sum of the vectors $2\hat{\imath}+4\hat{\jmath}-5\hat{k}$ and $\vec{c}=\lambda\hat{\imath}+2\hat{\jmath}+3\hat{k}$ is equal to $\sqrt{2}$ .Find the value of λ.
31. Let $\vec{a}=\hat{\imath}+4\hat{\jmath}+2\hat{k}$, $\vec{b}=3\hat{\imath}-2\hat{\jmath}+7\hat{k}$ and $\vec{c}=2\hat{\imath}-\hat{\jmath}+4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$, and $\vec{c}.\vec{d}=15$.
32. If $\vec{a}=2\hat{\imath}-\hat{\jmath}-2\hat{k}$ and $\vec{b}=7\hat{\imath}+2\hat{\jmath}-3\hat{k}$, then express $\vec{b}$ in the form of $\vec{b}=\vec{b_1}+\vec{b_2}$, where $\vec{b_1}$ is parallel to $\vec{a}$ and $\vec{b_2}$ is perpendicular to $\vec{a}$.

CASE STUDY BASED QUESTIONS

33. A farmer has a triangular land for agriculture. The sides are denoted by $\overrightarrow{AB}=3\hat{\imath}+\hat{\jmath}+5\hat{k}$, $\overrightarrow{BC}=-\hat{\imath}+2\hat{\jmath}-\hat{k}$ and $\overrightarrow{CA}=-2\hat{\imath}-3\hat{\jmath}-4\hat{k}$.

Using the information given above, answer the following :

(i) The value of $\angle$BCA in $\triangle$ABC is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

(ii) △ABC is

(a) isosceles triangle (b) right angled and isosceles triangle

(c) right angled and scalene triangle (d) equilateral triangle

(iii) $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA}$ equals to

(a) $\vec{0}$ (b) $-3\hat{\imath} + 2\hat{\jmath} - 4\hat{k}$ (c) $-\hat{\imath} - \hat{\jmath} - \hat{k}$ (d) $\hat{\imath} + 2\hat{\jmath} - \hat{k}$

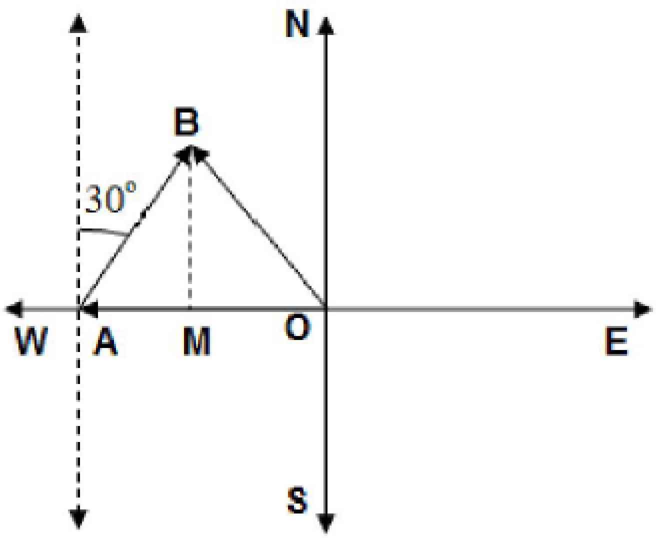
(iv) What is the area of △ABC?

(a) $\frac{\sqrt{147}}{2}$ sq.units (b) $\frac{\sqrt{17}}{2}$ sq.units (c) $\frac{\sqrt{174}}{2}$ sq.units (d) $\frac{\sqrt{471}}{2}$ sq.units

(v) Let a perpendicular is drawn from C on the side AB such that it meets AB at D. The length of this perpendicular CD is

(a) $\sqrt{\frac{174}{35}}$ sq.units (b) $\sqrt{\frac{74}{35}}$ sq.units (c) $\sqrt{\frac{174}{53}}$ sq.units (d) $\sqrt{\frac{147}{35}}$ sq.units

34. A girl walks 4 km towards west, then 3 km in a direction 30^0 east of north and then she stops. The situation has been depicted in the diagram as shown below, assuming that the girl starts her walk from O.



In the diagram, ON represents positive y-axis and North direction, OE represents positive x-axis and East direction. Similarly, OW is representing negative x-axis and West direction, whereas OS represents negative y-axis and South direction.

Let OA = 4 km, AB = 3 km.

Using the information given above, answer the following :

(i) What is the vector $\overrightarrow{OA}$?

(a) $4\hat{\imath}$ (b) $-4\hat{\imath}$ (c) $3\hat{\imath}$ (d) $3\hat{\jmath}$

(ii) What is the position vector of point B?

(a) $-\frac{5}{2}\hat{\imath}-\frac{3\sqrt{3}}{2}\hat{\jmath}$ (b) $\frac{5}{2}\hat{\imath}+\frac{3\sqrt{3}}{2}\hat{\jmath}$ (c) $-\frac{5}{2}\hat{\imath}+\frac{3\sqrt{3}}{2}\hat{\jmath}$ (d) $-\frac{5}{2}\hat{\imath}-\frac{3\sqrt{3}}{2}\hat{\jmath}$

(iii) What is the vector $\overrightarrow{AB}$

(a) $\frac{3}{2}\hat{\imath}+\frac{3\sqrt{3}}{2}\hat{k}$ (b) $-\frac{3}{2}\hat{\imath}+\frac{3\sqrt{3}}{2}\hat{\jmath}$ (c) $\frac{3}{2}\hat{\imath}-\frac{3\sqrt{3}}{2}\hat{\jmath}$ (d) $\frac{3}{2}\hat{\imath}+\frac{3\sqrt{3}}{2}\hat{\jmath}$

(iv) What is the value of $\overrightarrow{AB}\times\overrightarrow{OA}$?

(a) $6\sqrt{3}\,\hat{\imath}$ (b) $6\sqrt{3}\,\hat{k}$ (c) $-6\sqrt{3}\,\hat{k}$ (d) $\vec{0}$

(v) What is the ar (OAB)?

(a) $6\sqrt{3}$ (b) $3\sqrt{3}$ (c) $\sqrt{3}$ (d) $2\sqrt{3}$

Answers

1. $\frac{\pi}{3}$ **2.** $\frac{1}{3},\frac{2}{3},\frac{-2}{3}$ **3.** $|\vec{a}|=|\vec{b}|=3$ **4.** $\frac{5}{6}\sqrt{6}$ **5.** 4 **6.** $\frac{1}{3}(\hat{\imath}+2\hat{\jmath}-2\hat{k})$ **7.** 25 **8.** $\frac{5}{2}$ **9.** 5 sq.units **10.** $-\frac{1}{3}$ **11.** $(\alpha,-\beta,\ \gamma)$ **12.** $\vec{a}+2\vec{b}$ **13.** $3(\hat{\imath}-2\hat{\jmath}+2\hat{k})$ **14.** $\frac{1}{\sqrt{14}},\frac{2}{\sqrt{14}},\frac{3}{\sqrt{14}}$ **15.** $a=-1, b=1, a+b=0$ **16.** $\frac{1}{\sqrt{165}}(10\hat{\imath}+7\hat{\jmath}-4\hat{k})$ **17.** $\lambda=1$ **20.** $-\frac{1}{\sqrt{6}}\hat{\imath}+\frac{1}{\sqrt{6}}\hat{\jmath}-\frac{1}{\sqrt{6}}\hat{k}$ **21.** 4 **22.** $\frac{1}{7}(3\hat{\imath}-6\hat{\jmath}+2\hat{k}), 11\sqrt{5}$ sq. unit **23.** $\sqrt{2}$ **24.** $\vec{c}=\left(\frac{5}{3}\hat{\imath}+\frac{2}{3}\hat{\jmath}+\frac{2}{3}\hat{k}\right)$ **26.** $\lambda=2$ **27.** $-\frac{21}{2}$ **28.** $5\sqrt{2}$ **29.** $\frac{1}{2}\sqrt{210}$ **30.** $\lambda=$**1** **31.** $\frac{1}{3}(160\,\hat{\imath}-5\hat{\jmath}+70\,\hat{k})$ **32.** $\overrightarrow{b_1}=4\,\hat{\imath}-2\,\hat{\jmath}-4\,\hat{k},\ \overrightarrow{b_2}=3\,\hat{\imath}+4\,\hat{\jmath}+\hat{k},$

33. (i) d (ii) c (iii) a (iv) c (v) a **34.** (i) b (ii) c (iii) d (iv) b (v) b

THREE - DIMENSIONAL GEOMETRY

Summary

1. **Distance formula**: Distance between two points A(x_1, y_1, z_1) and B (x_2, y_2, z_2) is AB $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$
2. **Section formula**: Coordinates of a point P, which divides the line segment joining two given points A(x_1, y_1, z_1) and B(x_2, y_2, z_2) in the ratio *m:n*
   (i). internally, are $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}, \frac{mz_2+nz_1}{m+n}\right)$,
   (ii) externally, are $\left(\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}, \frac{mz_2-nz_1}{m-n}\right)$
   (iii) coordinate of mid-point of AB are $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$
3. **Direction cosines of a line** :
   (i) The direction of a line OP is determined by the angles α, β, γ which it makes with OX, OY,OZ respectively. These angles are called the direction angles and their cosines are called the direction cosines.
   (ii) Direction cosines of a line are denoted by *l, m, n* where $l = \cos\alpha$, $m = \cos\beta$, $n = \cos\gamma$
   (iii) $l^2+m^2+n^2=1$ i.e $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$
4. **Direction ratio of a line**: (i) Numbers proportional to the direction cosines of a line are called direction ratios of the line. If a ,b ,c are the direction ratios of the line, then
   $$\frac{l}{a} = \frac{m}{b} = \frac{n}{c}$$
   (ii) Direction cosines of the line are
   $$l = \frac{a}{\sqrt{a^2+b^2+c^2}}, \quad m = \frac{b}{\sqrt{a^2+b^2+c^2}}, \quad n = \frac{c}{\sqrt{a^2+b^2+c^2}}$$
   (iii) Direction ratios of a line AB passing through the points A$(x_1,y_1,z_1)$and B (x_2,y_2,z_2) are $x_2 - x_1, y_2 - y_1, z_2 - z_1$
5. **STRAIGHT LINE:**. (i) Vector equation of a Line passing through a point $\vec{a}$ and along the direction $\vec{b}$ is given by $\vec{r} = \vec{a} + \lambda\vec{b}$,
   (ii) Cartesian equation of a Line passing through the point (x_1, y_1, z_1) having direction ratios are *a,b,c* is given by
   $$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}.$$
   (iii) Vector equation of a Line passing through two points, with position vectors $\vec{a}$ and $\vec{b}$, is given by $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$
   (iv) Cartesian equation of a Line passing through the point (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by
   $$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$
6. (a) **Shortest distance between two skew- lines:**
   (i) If the Vector equations of two lines are: $\vec{r} = \overrightarrow{a_1} + \boldsymbol{\lambda}\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \boldsymbol{\mu}\overrightarrow{b_2}$, then the shortest distance between them is

$$d = \left|\frac{(\overrightarrow{a_2}-\overrightarrow{a_1}).(\overrightarrow{b_1}\times\overrightarrow{b_2})}{|\overrightarrow{b_1}\times\overrightarrow{b_2}|}\right|$$

If shortest distance between them is zero, then they intersect each other i.e., they are coplanar. Hence if above lines are coplanar then

$$(\overrightarrow{a_2}-\overrightarrow{a_1}).(\overrightarrow{b_1}\times\overrightarrow{b_2}) = 0$$

(ii) If the Cartesian equations of two lines are:

$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$, then the shortest distance between them is

$$d = \frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1c_2-b_2c_1)^2+(c_1a_2-c_2a_1)^2+(a_1b_2-a_2b_1)^2}}$$

If shortest distance between them is zero, then they intersect each other i.e., they are coplanar. Hence if above lines are coplanar then

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

(b) Shortest distance between two parallel lines: If two lines are parallel, then they are coplanar.

Let the lines be $\vec{r} = \overrightarrow{a_1} + \boldsymbol{\lambda}\vec{b}$, and $\vec{r} = \overrightarrow{a_2} + \boldsymbol{\mu}\vec{b}$, then the shortest distance between them is

$$d = \left|\frac{\vec{b}\times(\overrightarrow{a_2}-\overrightarrow{a_1})}{|\vec{b}|}\right|$$

7. **General equation of a plane in vector form** : It is given by $\vec{r}.\vec{n} = d$, where $\vec{n}$ is a vector normal to plane.
8. **General equation of a plane in Cartesian form : $\boldsymbol{Ax+By+Cz+D=0}$,** where A, B, C are the direction ratios of the normal to the plane.
9. **General equation of a plane passing through a point** :- If position vector of a given point on the plane is $\vec{a}$ then the equation of the plane is given by $(\vec{r}-\vec{a}).\vec{n} = 0$, where $\vec{n}$ is a vector perpendicular to the plane.

   If coordinates of the given point are (x_1, y_1, z_1) then equation of the plane is given by is

   $A(x-x_1)+B(y-y_1)+C(z-z_1) = 0$, where A, B, C are the direction ratios of the normal to the plane.

10. **Intercept form of equation of a plane :** The equation of a plane which cuts off intercepts a, b and c on x-axis, y-axis, z-axis respectively is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c} = 1$.
11. **Equation of a plane in normal form**: $\vec{r}.\hat{n}$ = p, where $\hat{n}$ is a unit vector along perpendicular from origin to the plane and 'p' is distance of plane from origin. p is always positive.

In Cartesian form, it is given by $lx + my + nz = p$, where l, m, n are the direction cosines of normal to the plane and 'p' is the distance of the plane from origin. p is always positive.

12. Equation of a plane passing through three non-collinear points :- If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of three non-collinear points, then equation of the plane through these three points is given by : $(\vec{r} - \vec{a}).\{(\vec{b} - \vec{a}) \times (\vec{c} - \vec{a})\} = 0$.

In Cartesian form, equation of the plane passing through three non-collinear points (x_1, y_1, z_1) , (x_2, y_2, z_2) and (x_3, y_3, z_3) is given by

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

13. General equation of a plane (vector form) passing through the line of the intersection of planes $\vec{r}.\vec{n_1} = d_1$ and $\vec{r}.\vec{n_2} = d_2$ is $\vec{r}.(\vec{n}_1 + \lambda\vec{n}_2) = d_1 + \lambda d_2$, where λ is a constant and can be calculated from given condition.
In Cartesian form, equation of the plane passing through the line of the intersection of the planes $A_1x+B_1y+C_1z+D_1 = 0$ and $A_2x+B_2y+C_2z+D_2 = 0$ is $(A_1x+B_1y+C_1z+D_1) +\lambda (A_2x+B_2y+C_2z+D_2) = 0$, where λ is a constant and can be calculated from given condition.

14. Distance of the plane(vector form) $\vec{r}.\vec{n} = d$, from a point with position vector $\vec{a}$, is $\left|\frac{\vec{a}.\vec{n} - d}{|\vec{n}|}\right|$.

In Cartesian form, Distance of the plane $Ax + By + Cz + D = 0$ from a point (x_1,y_1,z_1) is $\left|\frac{Ax_1 + By_1 + Cz_1 + D}{\sqrt{A^2+B^2+C^2}}\right|$.

SECTION – A (1 MARK)

1. Find the cartesian equation of the line which passes through the point $(-2, 4, -5)$ and is parallel to the line $\frac{x+3}{3} = \frac{4-y}{5} = \frac{z+8}{6}$
2. Find the distance of a point $(2,\ 5, -3)$ from the plane $\vec{r}.(6\hat{\imath} - 3\hat{\jmath} + 2\hat{k}) = 4$.
3. Write the direction ratios of the line : $3x + 1 = 6y - 2 = 1 - z$
4. If α, β, γ are the angles which a given line makes with positive direction of the axes, then prove that $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$
5. Write the equation of a plane which is at a distance of $5\sqrt{3}$ units from the origin and the normal to which is equally inclined to the coordinate axes.
6. If the cartesian equation of a line is $\frac{3-x}{5} = \frac{y+4}{7} = \frac{2z-6}{4}$, write the vector equation for the line.
7. Write the direction cosines of the normal to the plane $3x + 4y + 12z = 5\mathbf{2}$ **.**
8. Write the direction ratios of the following line: $x = -3,\ \frac{y-4}{3} = \frac{2-z}{1}$

9. Find the vector equation of a plane which is at a distance of 5 units from the origin and its normal vector is $2\,\hat{\imath} - 3\,\hat{\jmath} + 6\,\hat{k}$.

SECTION – B (2 MARKS)

10. If $P(2,3,4)$ is the foot of perpendicular from origin to a plane, then write the vector equation of this plane.
11. Write the sum of the intercepts cut off by the plane $\vec{r}.\left(2\,\hat{\imath} + \hat{\jmath} - \hat{k}\right) - 5 = 0$ on the three axes.
12. Show that the lines $\frac{x-1}{3} = \frac{y-1}{-1}, z + 1 = 0$ and $\frac{x-4}{2} = \frac{z+1}{3},\ y = 0$ intersect each other. Also find their point of intersection.
13. What is the distance of the point $(-1, -5, -10)$ from the point of intersection of the line $\vec{r} = 2\hat{\imath} - \hat{\jmath} + 2\hat{k} + \boldsymbol{\lambda}(3\hat{\imath} + 4\hat{\jmath} + 2\hat{k})$ and the plane $\vec{r}.\left(\hat{\imath} - \hat{\jmath} + \hat{k}\right) = 5$
14. If the point $(1,\ 1, p)$ and $(-3,\ 0, 1)$ be equidistant from the plane $\vec{r}.\left(3\,\hat{\imath} + 4\,\hat{\jmath} - 12\hat{k}\right) + 13 = 0$, then find the value of p.

SECTION – C (3 MARKS)

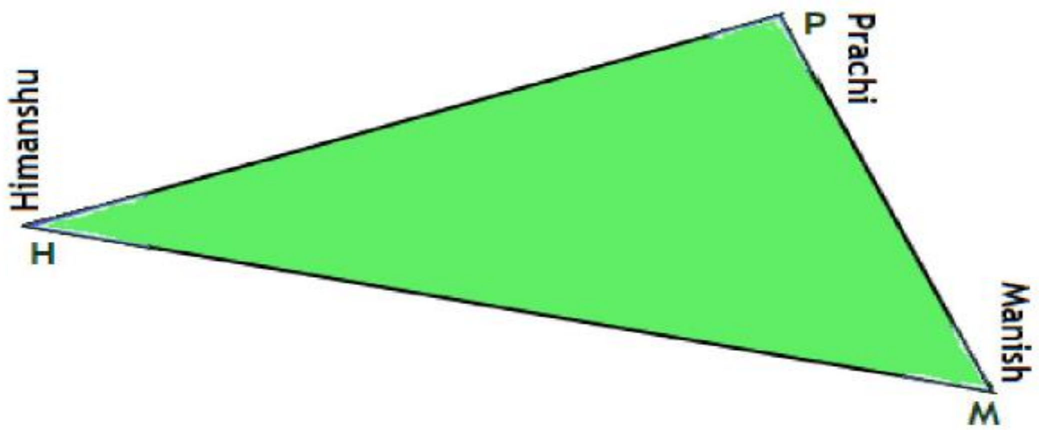
15. Find the vector equation of the plane through the intersection of the planes $\vec{r}\ .\left(\hat{\imath} + \hat{\jmath} + \hat{k}\right) - 6 = 0$ and $\vec{r}.\ \left(2\,\hat{\imath} + 3\,\hat{\jmath} + \ 4\,\hat{k}\right) = -5$ and the point $(\,1\,, 1\,, 1)$.
16. Find the shortest distance between the following pair of lines :
$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}; \qquad \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$$
17. Find the equation of the plane (s) passing through the intersection of the planes $x + 3y + 6 = 0$ and $3x - y - 4\,z = 0$ and whose perpendicular distance from the origin is unity.
18. Find the distance of the point $3\hat{\imath} - 2\,\hat{\jmath} + \hat{k}$ from the plane $3x + y - z + 2 = 0$ measured parallel to the line $\frac{x-1}{2} = \frac{y+2}{-3} = \frac{z-1}{1}$.
19. Find the co-ordinates of the point where the line $\vec{r} = \left(-\,\hat{\imath} - 2\,\hat{\jmath} - 3\,\hat{k}\right) + \lambda\left(3\,\hat{\imath} + 4\,\hat{\jmath} + 3\,\hat{k}\right)$ meets the plane which is perpendicular to the vector $\vec{n} = \hat{\imath} + \ \hat{\jmath} + 3\,\hat{k}$ and at a distance of $\frac{4}{\sqrt{11}}$ from the origin.
20. Find the equation of the plane passing through the point $(1, 2, 1)$ and is perpendicular to the line joining the points $(1, 4, 2)$ and $(2, 3, 5)$. Also, find the perpendicular distance of the plane from the origin.
21. Show that the lines $\vec{r} = (-3\,\hat{\imath} + \hat{\jmath} + 5\,\hat{k}\,) + \lambda\left(-3\hat{\imath} + \hat{\jmath} + \ 5\hat{k}\right)andd\vec{r} = \left(-\,\hat{\imath} + 2\hat{\jmath} + 5\hat{k}\right) + \mu\left(-\,\hat{\imath} + 2\hat{\jmath} + 5\hat{k}\right)$ are coplanar. Also find the Cartesian and vector equation of the plane containing these lines.
22. Find the equation of the plane which contains the intersection of the planes $\vec{r}\ .\left(\hat{\imath} - 2\,\hat{\jmath} + 3\hat{k}\right) - 4 = 0$ and $\vec{r}.\left(-2\,\hat{\imath} + \ \hat{\jmath} + \ \hat{k}\right) + 5 = 0$ and whose intercept on x- axis is equal to that of on y-axis.

SECTION – D (5 MARKS)

23. Find the equation of the plane through the points A $(3,\ 2,1)$, B $(4,\ 2,-2)$ and C $(6,\ 5,-1)$ and hence find the value of λ for which A $(3,\ 2,1)$,B $(4,\ 2,-2)$, C $(6,\ 5,-1)$ and D$(\lambda,\ 5,5)$ are coplanar.
24. Find the shortest distance between the lines whose vector equations are: $\vec{r} = (1-t)\hat{\imath} + (t-2)\hat{\jmath} + (3-2t)\hat{k}$ and $\vec{r} = (s+1)\hat{\imath} + (2s-1)\hat{\jmath} - (2s+1)\hat{k}$
25. Find the coordinates of the foot of the perpendicular and perpendicular distance of the point $(1,3,4)$ from the plane $2x - y + z + 3 = 0$. Find also, the image of the point in the plane.

. CASE STUDY BASED QUESTIONS

26. Three students Manish, Himanshu and Prachi were sitting in the playground, in such a way that they were not along a line (i.e., the points of their sitting positions are non-collinear). If the points at which they were sitting can be expressed in terms of position vectors M$(2\hat{\imath} - \hat{\jmath} + \hat{k}\ )$, H$(\hat{\imath} - 3\hat{\jmath} - 5\hat{k})$ and P$(3\hat{\imath} - 4\hat{\jmath} - 4\hat{k})$, then answer the following questions based on these facts :



(i) The triangular region formed by the given position vectors is

(a) equilateral (b) isosceles (c) right angled (d) right angled as well as isosceles

(ii) If the triangular region formed is right angled type, then name of the students, who may be sitting in
the position of hypotenuse of the triangle, are

(a) Himanshu, Manish (b) Manish, Prachi (c) Prachi, Himanshu
(d) not possible, as the triangle is not right angled

(iii) Equation of plane in which the three students are sitting, is

(a) $8x + 11y - 5z = 10$ (b) $8x + 11y - 5z = -10$ (c) $8x - 11y - 5z = 0$
(d) $8x + 11y - 5z = 0$

(iv) Normal to the plane obtained in (iii), is

(a) $2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ (b) $8\hat{\imath} + 11\hat{\jmath} - 5\hat{k}$ (c) $8\hat{\imath} - 11\hat{\jmath} - 5\hat{k}$ (d) $-8\hat{\imath} - 11\hat{\jmath} - 5\hat{k}$

(v) What are the direction cosines of the normal to the plane obtained in (iii)?

(a) $\frac{8}{\sqrt{210}}, \frac{11}{\sqrt{210}}, \frac{-5}{\sqrt{210}}$ (b) $\frac{8}{\sqrt{210}}, \frac{-11}{\sqrt{210}}, \frac{-5}{\sqrt{210}}$ (c) $\frac{-8}{\sqrt{210}}, \frac{-11}{\sqrt{210}}, \frac{5}{\sqrt{210}}$ (d) none of these

27. A butterfly is moving in a straight path in the space. Let this path be denoted by a line l whose equation is $\frac{x-1}{2} = \frac{2-y}{3} = \frac{z-3}{4}$

Using the information given above, answer the following with reference to the line :

(i) The position vector of the point on the line is

(a) $\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ (b) $\hat{\imath} + 2\hat{\jmath} + \hat{k}$ (c) $2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$ (d) $2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$

(ii) What are the direction ratios of the line?

(a) $2, 3, 4$ (b) $-2, 3, 4$ (c) $2, -3, 4$ (d) $2, 3, -4$

(iii) If the z-coordinate of a point on this line is 11, then the x-coordinate of the same point on this line, is

(a) -5 (b) 5 (c) 0 (d) 1

(iv) The vector equation of the given line is

(a) $\vec{r} = (\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) + \lambda(2\hat{\imath} - 3\hat{\jmath} + 4\hat{k})$

(b) $\vec{r} = (\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) + \lambda(2\hat{\imath} + 3\hat{\jmath} + 4\hat{k})$

(c) $\vec{r} = (\hat{\imath} - 2\hat{\jmath} + 3\hat{k}) + \lambda(2\hat{\imath} - 3\hat{\jmath} + 4\hat{k})$

(d) $\vec{r} = (\hat{\imath} + 2\hat{\jmath} - 3\hat{k}) + \lambda(2\hat{\imath} - 3\hat{\jmath} + 4\hat{k})$

(v) The unit vector in the direction of the vector parallel to the given line, is

(a) $\frac{\hat{\imath}+2\hat{\jmath}+3\hat{k}}{\sqrt{14}}$ (b) $\frac{2\hat{\imath}+3\hat{\jmath}+4\hat{k}}{\sqrt{29}}$ (c) $\frac{\hat{\imath}-2\hat{\jmath}+3\hat{k}}{\sqrt{14}}$ (d) $\frac{2\hat{\imath}-3\hat{\jmath}+4\hat{k}}{\sqrt{29}}$

28. A bird is located at the point A(3, 2, 8) in space. It wants to reach to the plane whose equation is given by $3x + 2y + 6z - 12 = 0$ in the shortest time.

Using the information given above, answer the following :

(i) The normal to the plane is given by

(a) $-3\hat{\imath} - 2\hat{\jmath} + 6\hat{k}$ (b) $-3\hat{\imath} + 2\hat{\jmath} - 6\hat{k}$ (c) $3\hat{\imath} + 2\hat{\jmath} + 6\hat{k}$ (d) $-\hat{\imath} - \hat{\jmath} - \hat{k}$

(ii) The direction cosines of the normal to the plane are given by

(a) $\frac{6}{7}, \frac{3}{7}, \frac{2}{7}$ (b) $\frac{3}{7}, \frac{2}{7}, \frac{6}{7}$ (c) $\frac{-3}{7}, \frac{-2}{7}, \frac{-6}{7}$ (d) $\frac{3}{7}, \frac{2}{7}, \frac{-6}{7}$

(iii) What is the distance of given plane from (0, 0, 0)?

(a) $\frac{3}{7}$ (b) $\frac{12}{7}$ (c) $\frac{16}{7}$ (d) $\frac{4}{7}$

(iv) The distance covered by the bird to reach the plane in shortest time is

(a) 7 units (b) 3 units (c) 12 units (d) 7 units

(v) Find the point at which the bird must land on the plane, in the shortest time.

(a) (3, 2, 0) (b) (0, 0, 2) (c) (2, 0, 0) (d) (0, 2, 0)

Answers

1. $\frac{x+2}{3} = \frac{y-4}{-5} = \frac{z+5}{6}$ **2.** $\frac{13}{7}$ **3.** $2, 1, -6$ **5.** $x + y + z = 15$

6. $\vec{r} = (3\hat{\imath} - 4\hat{\jmath} + 3\hat{k}) + \lambda(-5\hat{\imath} + 7\hat{\jmath} + 2\hat{k})$ **7.** $\frac{3}{13}, \frac{4}{13}. \frac{12}{13}$ **8.** $0, 3, -1$

9. $\vec{r}.\left(\frac{2}{7}\hat{\imath} - \frac{3}{7}\hat{\jmath} + \frac{6}{7}\hat{k}\right) = 5$ **10.** $\vec{r}.(2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}) = 29$ **11.** $\frac{5}{2}$ **12.** $(4, 0, -1)$

13. 13 **14.** $p = 1\ or\ \frac{7}{3}$ **15.** $\vec{r}.\left(20\hat{\imath} + 23\hat{\jmath} + 26\hat{k}\right) = 69$ **16.** $\frac{1}{\sqrt{6}}$

17. $2x + y - 2z + 3 = 0$ & $x - 2y - 2z + 3 = 0$ **18.** $4\sqrt{14}$ unit **19.** $(2,\ 2,\ 0)$

20. $x - y + 3z - 2 = 0,\ \frac{2\sqrt{11}}{11}$ **21.** $x - 2y + z = 0$ or $\vec{r}.\left(\hat{\imath} - 2\hat{\jmath} + \hat{k}\right) = 0$ **22.** $x + y - 4z = 1$ **23.** $9x - 7y + 3z - 16 = 0,\ \lambda = 4$ **24.** $\frac{\mathbf{8\sqrt{29}}}{\mathbf{29}}$ **25.** $(-1,4,3)$, $\sqrt{6}$, $(-3,5,2)$

26. (i) c (ii) a (iii) d (iv) b (v) a **27.** (i) a (ii) c (iii) b (iv) a (v) d

28. (i) c (ii) b (iii) b (iv) d (v) b

LINEAR PROGRAMMING PROBLEMS

LINEAR PROGRAMMING PROBLEM (LPP)

A linear programming problem deals with the optimization (maximization/minimization) of a linear function of two variables(say x and y) which is known as objective function subject to :

i) The variables x and y are nonnegative
ii) The variables x and y satisfy a set of linear inequalities which are called linear constraints.

OBJECTIVE FUNCTION

A linear function z = ax+by where a and b are constants which has to be maximized or minimized is called a linear objective function.

DECISION VARIABLES

In the objective function z = ax+by, x and y are called decision variables.

CONSTRAINTS

The linear inequalities or restrictions on the variables of an LPP are called constraints. The conditions $x \geq 0$,$y \geq 0$ are called non-negative constraints.

FEASIBLE REGION

The common region determined by all the constraints including non-negative constraints $x \geq 0$,$y \geq 0$ of an LPP is called the feasible region for the problem.

FEASIBLE SOLUTIONS

Points within and on the boundary of the feasible region for an LPP represent feasible solutions.

INFEASIBLE SOLUTIONS

Any point outside feasible region is called infeasible region.

OPTIMAL(FEASIBLE) SOLUTION

Any point in the feasible region that gives the optimal value (maximum or minimum) of the objective function is called an optimal solution.

THEOREM-1

Let R be the feasible region (convex polygon) for an LPP and Z=ax+by be the objective function. When Z has an optimal value (maximum or minimum) where x and y are subject to constraints described by linear inequalities, this optimal value must occur at a corner point(vertex) of the feasible region.

THEOREM-2

Let R be the feasible region for a LPP and Z = ax+by be the objective function . If R is bounded , then the objective function Z has both a maximum and a minimum value on R and each of these occur at a corner point of R.

If the feasible region R is unbounded , then a maximum or a minimum value of the objective function may or may not exist.If it exists, it must occur at a corner point of R.

CORNER POINT METHOD FOR SOLVING A LPP

The method comprises of the following steps:

1) Find the feasible region of the LPP and determine its corner points(vertices) either by inspection or by solving the two equations of the lines intersecting at the point.
2) Evaluate the objective function Z = ax+by at each corner point.
   Let M and m respectively denote the largest and the smallest values of Z.

3).i) When the feasible region is bounded , M and m are respectively the maximum and minimum values of Z.

ii) In case the feasible region is unbounded

a) M is maximum value of Z , if the open half plane determined by ax+by> M has no point in common with the feasible region. Otherwise Z has no maximum value.
b) Similarly , m is minimum value of Z , if the open half plane determined by ax+by< m has no common point with the feasible region. Otherwise Z has no minimum value.

MULTIPLE OPTIMAL POINTS

If two corner points of the feasible region are optimal solutions of the same type i.e. both produce the same maximum or minimum , then any point on the line segment joining these two points is also an optimal solution of the same type.

5 MARKS QUESTIONS

1.Solve the following linear programming problem graphically:

Maximise Z = 3x+4y

Subject to the constraints

$x+y\leq 4$

$x\geq 0\,, y\geq 0$

Ans:Maximum value of Z is 16 at the point (0,4)

2. Solve the following linear programming problem graphically:

Maximise Z = 4x+y

Subject to the constraints

$x+y \leq 50$

$3x+y \leq 90$

$x \geq 0, y \geq 0$

Ans: Maximum value of Z is 120 at the point (30,0)

1. Maximize: Z = x+2y

Subject to the constraints

$x+2y \geq 100$

$2x-y \leq 0$

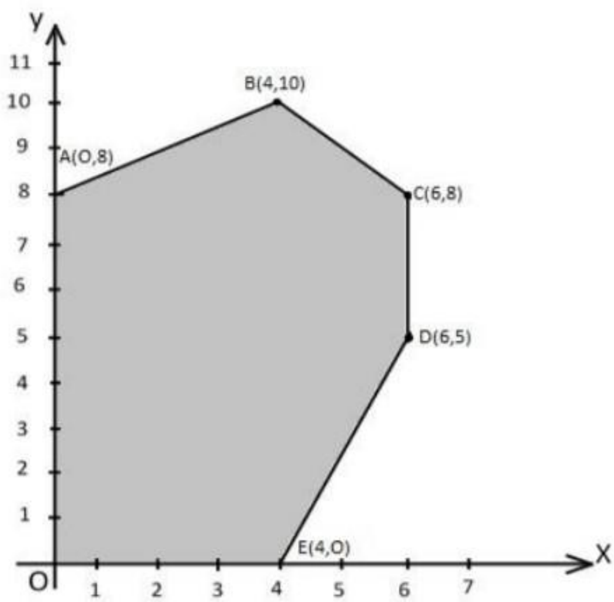
$2x+y \leq 200$

$x \geq 0, y \geq 0$

Solve the above LPP graphically.

Ans: Maximum value of Z is 400 at the point (0,200)

2. The corner points of the feasible region determined by the system of linear constraints are as shown below:



Answer each of the following:

i) Let Z = 3x-4y be the objective function. Find the maximum and minimum value of Z and also the corresponding points at which the maximum and minimum value occurs.

ii) Let Z = px+qy , where p,q>0 be the objective function. Find the condition on p and q so that the maximum value of occurs at B(4,10) and C(6,8).Also mention the number of optimal solutions in this case.

Ans: i) Maximum Z = 12 at (4,0) and Minimum Z = -32 at (0,8) ii) Number of optimal solutions are infinite

3. Solve the following linear programming problem graphically:

Maximise Z = 3x+9y

Subject to the constraints

$x+3y \leq 60$

$x+y \geq 10$

$x \leq y$

$x \geq 0 , y \geq 0$

Ans: Maximum value of Z occurs at two corner points (15,15) and (0,20) and maximum value is 180 in each case.

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PROBABILITY

CONDITIONAL PROBABILITY

The conditional probability of an event E ,given the occurrence of the event F is given by

$$P(E/F) = \frac{P(E\cap F)}{P(F)} , \quad P(F) \neq 0$$

$0 \leq P(E/F) \leq 1$

$P(E'/F) = 1 - P(E/F)$

$P((E \cup F)/G) = P(E/G) + P(F/G) - P((E\cap F)/G)$

MULTIPLICATION THEOREM OF PROBABILITY

$P(E\cap F) = P(E).P(F/E) , \quad P(E) \neq 0$

$P(E\cap F) = P(F).P(E/F) , \quad P(F) \neq 0$

INDEPENDENT EVENTS

If E and F are independent events then

$P(E/F)= P(E) , \quad P(F) \neq 0$

$P(F/E) =P(F) , \quad P(F) \neq 0$

$P(E\cap F) = P(E).P(F)$

THEOREM OF TOATL PROBABILITY

Let $\{E_{1,}E_{2,}E_3 \dots\dots\dots\dots\dots\dots E_n\}$ be a partition of a sample space and suppose that each of $E_{1,}E_{2,}E_3 \dots\dots\dots\dots\dots\dots E_n$ has non zeroprobability.Let A be any event associated with S . Then

$P(A) = P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + \dots\dots\dots + P(E_n)P(A/E_n)$

BAYE'S THEOREM

If $E_{1,}E_{2,}E_3 \dots\dots\dots\dots\dots\dots E_n$ are events which constitute a partition of a sample space S i.e. $E_{1,}E_{2,}E_3 \dots\dots\dots\dots\dots\dots E_n$ are pairwise disjoint and $E_1UE_2UE_3U \dots\dots\dots\dots\dots\dots UE_n$ = S and A be any event with nonzero probability then

$$P(E_i/A) = \frac{P(E_i)P(A/E_i)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + \cdots \dots\dots\dots\dots + P(E_n)P(A/E_n)}$$

RANDOM VARIABLE

A random variable is a real valued function whose domain is sample space of a random experiment.

The probability distribution of a random variable X is system of numbers

X	x_1	x_2	....	x_n
P(X)	p_1	p_2	....	p_n

Where $p_i > 0$ and $\sum_{i=1}^{n} p_i = 1$, i= 1,2,..........n

1 MARK QUESTIONS

1. If P(A) = $\frac{1}{2}$, P(B) = 0, what is P(A/B) ? Ans-not defined
2. Compute P(A/B) if P(B) =0.5 and P(A∩ B) = 0.32. Ans:$\frac{16}{25}$
3. If P(A) = $\frac{6}{11}$, P(B) = $\frac{5}{11}$ and P(AUB) = $\frac{7}{11}$ find P(B/A). Ans:$\frac{2}{3}$
4. Evaluate P(AUB) if 2P(A) = P(B) =$\frac{5}{13}$ and P(A/B) = $\frac{2}{5}$. Ans: $\frac{11}{26}$
5. If A and B are events such that P(A/B) = P(B/A) and P(A) = $\frac{1}{12}$ find P(B) .
   Ans:$\frac{1}{12}$
6. Given that the events A and B are such that P(A) = $\frac{1}{2}$, P(AUB)=$\frac{3}{5}$ and P(B) =p . Find p if A and B are
   i). mutually exclusive ii). Independent
   Ans-i) p= 1/10 ii) p=1/5
7. Let A and B be the events with P(A) = $\frac{3}{5}$, P(B)=$\frac{3}{10}$ and P(A∩ B) =$\frac{1}{5}$. Are A and B independent ?
   Ans- A and B are not independent.
8. If P(A) =0.4 , P(B) = p, P(A∩ B) = 0.1 . Find the value of p if A and B are independent events. Ans- p=0.25
9. If A and B are independent events find P(B) if P(AUB) = 0.60 and P(A) = 0.35.
   Ans: $\frac{5}{13}$
10. Given two independent events A and B such that P(A) = 0.3 , P(B) = 0.6. Find P(neither A nor B).
    Ans: 0.18

2 MARKS QUESTIONS

1. **G**iven that E and F are independent events such that P(E) = 0.8, P(F) = 0.7 and P(E∩ F) = 0.6. Find P(E'/F').
   Ans- 0.2
2. If A and B are two events such that P(A)= $\frac{1}{4}$, P(B) =$\frac{1}{2}$ and P(A∩ B)= $\frac{1}{8}$. Find P(notA and notB). Ans- 1/8
3. If a die is thrown and a card is selected at random from a deck of 52 cards , what is the probability of getting an even number and a spade card ?
   Ans: $\frac{1}{8}$

4. A die marked 1,2,3 in red and 4,5,6 in green is thrown. Let A be the event 'the number is even' and B be the event 'the number is red'. Are A and B independent ?
Ans-No

5. A coin is biased so that the head is 3 times as likely to occur as tail.If a coin is tossed twice , find the probability distribution of number of tails.
Ans:

X	0	1	2
P(X)	9/16	6/16	1/16

6. The probability that it will rain on any particular day is 50%. Find the probability that it rains only on first four days of the week. Ans: $\frac{1}{128}$

7. The probability of finding a green signal on a busy crossing X is 30%. What is the probability of finding a green signal on X on two consecutive days out of three?
Ans: 63/500

8. Find k for the following probability distribution.

X	0	1	2	3
P(X)	k	k^2	k	0.04

Ans: 0.4

9. A bag contains 5 white, 7 red and 3 black balls. If three balls are drawn one by one without replacement, find the probability that none is red .Ans-8/65

3 MARKS QUESTIONS

1. A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from the first bag. Ans-2/3
2. A box has 5 blue and 4 red balls. One ball is drawn at random and not replaced. Its colour is also not noted. Then another ball is drawn at random. What is the probability of second ball being blue? Ans: 5/9
3. Suppose a girl throws a die. If she gets a 5 or 6, she tosses a coin three times and notes the number of heads. If she gets 1, 2, 3 or 4 she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one head, what is the probability that she threw 1, 2,3 or 4 with the die. Ans-8/11
4. There are three coins . One is a two headed coin(having head on both faces) , another is a biased coin that comes up heads 75% of the time and third is an unbiased coin.One of the three coins is chosen at random and tossed , it shows head, what is the probability that it was two headed coin ? Ans-4/9
5. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accident are 0.01,0.03 and 0.15 respectively.One of the insured persons meets with an accident, What is the probability that he is a scooter driver ? Ans-1/52

6. Two cards are drawn simultaneously (or successively without replacement) from a well shuffled pack of 52 cards. Find the probability distribution ofgetting the number of kings.

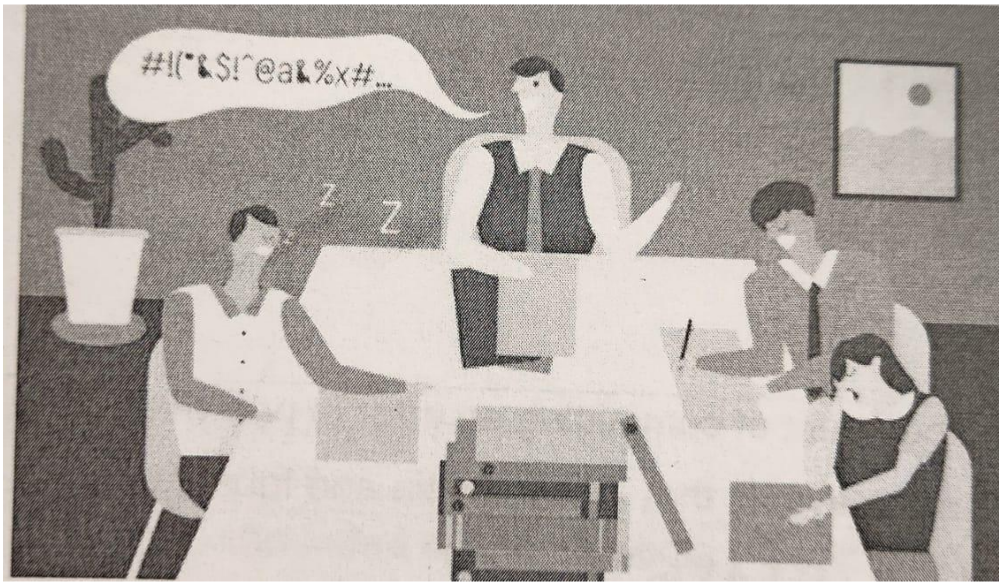
   Ans:

X	0	1	2
P(X)	188/221	32/221	1/221

7. A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six. Ans- 3/8
8. A letter is known to have come from either LONDON or CLIFTON. On the envelope just two consecutive letters ON are visible. What is the probability that the letter is come from LONDON? Ans- 12/17

CASE STUDY QUESTIONS

1. In an office three employees Vishal, Ranjan and Ankit process incoming copies of a certain forms. Vishal processes 50% of the forms, Ranjan processes 20% of the forms and Ankit processes the remaining 30% of the forms . Vishal has an error rate of 0.06, Ranjan has an error rate of 0.04 and Ankit has an error rate of 0.03.



Based on the above information answer the following:

i). The conditional probability that an error is committed in processing given that Ranjan processed the form is

a) 0.0210

b) 0.04

c) 0.47

d) 0.06

(ii) The probability that Ranjan processed the form and committed an error is :

a) 0.005
b) 0.006
c) 0.008
d) 0.68

(iii) The total probability of committing an error in processing the form is

a) 0
b) 0.047
c) 0.234
d) 1

(iv)The manager of the company wants to do a quality check . During inspection he selects a form at random from the days output of processed forms. If the form selected at random has an error , the probability that the form is not processed by Vishal is :

a) 1
b) 30/47
c) 20/47
d)17/47

(v) Let A be the event of committing an error in processing the form and let $E_{1,}E_2\ and\ E_3$ be the events that Vishal, Ranjan and Ankit processed the form. The value of $\sum_{i=1}^{3} P(E_i/A)$ is

a) 0
b) 0.03
c) 0.06
d 1

Ans: i) b ii) c iii) b iv) d v) d

2. Let X denote the number of college where you will apply after yours result and P(X=x) denotes your probability of getting admission in x number of college.It is given that

$$P(X=x) = \begin{cases} kx\ , if\ x = 0\ or\ 1 \\ 2kx\ ,\quad if\ x = 2 \\ k(5-x), if\ x = 3\ or\ 4 \\ 0, \quad if\ x > 4 \end{cases}$$

Where k is a positive constant.

Based on the above information answer the following:

i) The value of k is
a) 1 b) 1/3 c) 1/7 d) 1/8

ii) The probability you will get admission in exactly one college is
a) 1/2 b) 1/3 c) 1/8 d) 1/5

iii) The probability you will get admission in at most two colleges is
a) 7/12b) 5/8 c) 5/21 d) 8/17

iv) The probability you will get admission in at least two colleges is
a) 1/3 b) 2/7 c) 3/8 d) 7/8

v) The probability you will get admission in more than 4 colleges is
a) 0 b) 1/3 c) 1/2 d) 1/8

Ans- i) d ii) c iii) b iv) d v) a

3. Given three identical boxes 1^{st} ,2^{nd} and 3^{rd} each containing two coins. In 1^{st} box both coins are gold coins, in 2^{nd} box both are silver coins and in 3^{rd} box there is one gold and one silver coin. A person chooses a box at random and takes out a coin.

On the basis of the above information , answer the following questions:

i) The probability of choosing 1^{st} box is
 a) 1/6 b) ¼ c) 1/3 d) ½

ii) 1/2 b) 1 c) 1/3 d) 1 The probability of getting gold coin from 3^{rd} box is
 a) /6

iii) The probability of choosing 3^{rd} box and getting silver coin is
 a) 1/6 b) ¼ c) 1/3 d) ½

iv) The total probability of drawing gold coin is
 a) 1/6 b) ¼ c) 1/3 d) ½

v) If drawn coin is gold the probability that other coin in the box is also of gold is
 a) 1/3b) 2/3 c) 1/2 d) ¾

Ans- i) c ii) a iii) a iv) d v) b

5 - SAMPLE PAPERS

Kendriya Vidyalaya Sangathan Raipur -Region

Time: 3 Hours **Sample Question Paper- I (2020 – 21)** M. M: 80

Class-XII

Subject: - Mathematics

General Instructions:--

1. This question paper contains two **Parts A** and **B**. Each part is compulsory. Part **A** carries **24** marks and Part **B** carries **56** marks.
2. **Part-A** has Objective Type Questions and **Part-B** has Descriptive Type Questions
3. Both Part A and Part B have choice.

Part-A

1. It consists of two sections- **I** and **II**
2. Section **I** comprises of **16** very short answer type questions.
3. Section **II** contains **2** case studies. Each case study comprises of 5 case –based MCQs. An examinee is to attempt **any 4 out of 5 MCQs.**

Part-B

1. It consists of three sections- **III, IV** and **V.**
2. Section **III** comprises of 10 questions of **2 marks** each.
3. Section **IV** comprises of 7 questions of **3 marks** each.
4. Section **V** comprises of 3 questions of **5 marks** each.
5. Internal choice is provided in **3** questions of section- III, **2** questions of section- IV and **3** questions of section- V. You have to attempt only one of the alternatives in all such questions.

[Part – A]

SECTION- I[1 mark each]

1. Check whether the function f **:** Z → Z defined as $f(x) = x^2 + 2$ is one-one or not **Ans: No**

(OR)

If $R = \{(x, y) : x + 2y = 8\}$ is a relation on N, write the range of R. **Ans:** $\mathbf{R_R = \{1, 2, 3\}}$

2. A relation R in the set A $= \{1, 2, 3\}$ is defined as R= $\{(1,1), (1,2), (2,2), (3,3)\}$.Which element(s) of relation R be removed to make R an equivalence relation ? **Ans:** $\mathbf{(1, 2)}$

3. A relation R in the set of real numbers R defined as R $= \{(x, y): \sqrt{x} = y\}$ is a function or not. Justify **Ans: No**

(OR)

Find the domain of the function $f(x) = \frac{x^2+2x+1}{x^2-8x+12}$ **Ans:** $\mathbf{D_f = R - \{2, 6\}}$

4. If A and B are matrices of order $3 \times n$ and $m \times 5$ respectively, then find the order of the matrix $7A - 5B$, given that it is defined. **Ans: order of** $\mathbf{7A - 5B = 3 \times 5}$

5. Construct a 2×2 matrix, A=$[a_{ij}]$whose elements are given by $a_{ij} = \begin{cases} 1 & \text{if } i \neq j \\ 0 & \text{if } i = j \end{cases}$ **Ans:** $\begin{bmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} \end{bmatrix}$

(OR)

Given that A is a square matrix of order 3 and $|A| = -4$, find| $a$dj A | = 64 **Ans: 16**

6. Let $A=[a_{ij}]$ be a square matrix of order 3 and $|A| = -7$. .Find the value of $a_{11}\,A_{21} + a_{12}\,A_{22} + a_{13}\,A_{23}$ where A_{ij} is the cofactor of element a_{ij} **Ans: 0**

7. Evaluate : $\int e^x.(1 - \cot x + \operatorname{cosec}^2 x)dx$ **Ans:** $\boldsymbol{e^x(1 - \cot x) + C}$

(**OR**)

Evaluate : $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^2 \sin x\, dx$ **Ans: 0**

8. Find the area bounded by $y = x^2$, the x-axis and the lines $x = -1$ and $x = 1$ **Ans:** $\boldsymbol{\frac{2}{3}}$ **sq units**

9. How many arbitrary constants are there in the particular solution of differential equation $\frac{dy}{dx} = -3xy^2$; $y(0) = 1$ **Ans: No**

(**OR**)

For what value of n is the following a homogeneous differential equation: $\frac{dy}{dx} = \frac{x^3 - y^n}{x^2y + xy^2}$ **Ans: 3**

10. Find a unit vector in the direction opposite to $-\frac{3}{4}\hat{j}$ **Ans:** $\hat{j}$

11. Find the area of the triangle whose two sides are represented by the vectors $2\hat{\imath}$ and $-3\hat{j}$ **Ans: 3 sq.uni**

12. Write the projection of vector $2\,\hat{\imath} + 3\,\hat{j} - \hat{k}$ along the vector $\hat{\imath} + \hat{j}$. **Ans:** $\boldsymbol{\frac{5}{\sqrt{2}}}$

13. Find the direction cosines of the normal to YZ plane. Ans: $1, 0, 0$.

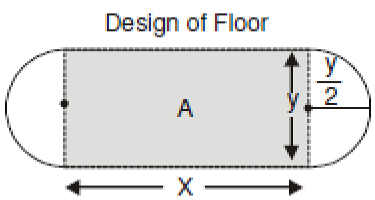
14. Write the direction ratios of the line : $3x + 1 = 6y - 2 = 1 - z$ **Ans:** $\boldsymbol{2, 1, -6}$

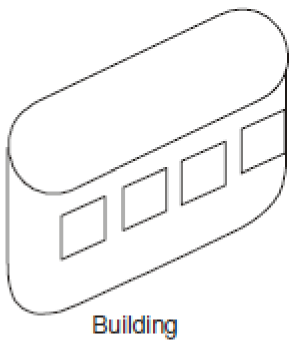
15. If A and B are two independent events with $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{4}$, then find $P(B'/A)$ **Ans:** $\boldsymbol{\frac{3}{4}}$

16. If a leap year is selected at random, then what is the chance that it will contain 53 Tuesday? **Ans:** $\boldsymbol{\frac{2}{7}}$

SECTION- II For Q.17 and Q18 attempt any four (MCQs) [1 mark each]

17. An architect designs a building for a multi-national company. The floor consists of a rectangular region with semicircular ends having a perimeter of 200 m as shown below





Based on the above information answer the following:

(i) If x and y represents the length and breadth of the rectangular region, then the relation between the variables is

(a) $x + \pi y = 100$ **(b)** $2x + \pi y = 200$ **(c)** $\pi x + y = 50$ **(d)** $x + y = 100$ **Ans: (b)**

(ii) The area of the rectangular region A expressed as a function of x is

(a) $\frac{2}{\pi}(100 - x^2)$ **(b)** $\frac{1}{\pi}(100 - x^2)$ **(c)** $\frac{x}{\pi}(100 - x)$ **(d)** $\pi y^2 + \frac{2}{\pi}(100 - x^2)$ **Ans: (a)**

(iii) The maximum value of area A is

(a) $\frac{\pi}{3200}\ m^2$ **(b)** $\frac{3200}{\pi}\ m^2$ **(c)** $\frac{5000}{\pi}\ m^2$ **(d)** $\frac{1000}{\pi}\ m^2$ **Ans: (c)**

(iv) The CEO of the multi-national company is interested in maximizing the area of the whole floor including the semi-circular ends. For this to happen the value of *x* should be **Ans: (a)**

(a) $0\ m$ **(b)** $30\ m$ **(c)** $50\ m$ **(d)** $80\ m$

(v) The extra area generated if the area of the whole floor is maximized is **Ans: (d)**

(a) $\frac{3000}{\pi}\ m^2$ **(b)** $\frac{5000}{\pi}\ m^2$ **(c)** $\frac{7000}{\pi}\ m^2$ **(d)** No change both areas are equal.

18. In an office three employees Vinay, Sonia and Iqbal process incoming copies of a certain form. Vinay process 50% of the forms. Sonia processes 20% and Iqbal the remaining 30% of the forms. Vinay has an error rate of 0.06, Sonia has an error rate of 0.04 and Iqbal has an error rate of 0.03.

Based on the above information answer the following:

(i) The conditional probability that an error is committed in processing given that Sonia processed the form is **(a)** 0.21 **(b)** 0.04 **(c)** 0.47 **(d)** 0.06 **Ans: (b)**

(ii) The probability that Sonia processed the form and committed an error is

(a) 0.005 **(b)** 0.006 **(c)** 0.008 **(d)** 0.68 **Ans: (c)**

(iii) The total probability of committing an error in processing the form is

(a) 0 **(b)** 0.47 **(c)** 0.234 **(d)** 1 **Ans: (b)**

(iv) The manager of the company wants to do quality check. During inspection he selects a form at random from the days output of processed forms. If the form selected at random has an error, the probability that the form is NOT processed by Vinay is

(a) 1 **(b)** $\frac{30}{47}$ **(c)** $\frac{20}{47}$ **(d)** $\frac{17}{47}$ **Ans: (d)**

(v) Let *A* be the event of committing an error in processing the form and let E_1, E_2 and E_3 be the events that Vinay, Sonia and Iqbal processed the form. The value of $\sum_{i=1}^{3} P(E_i/A)$

(a) 0 **(b)** 0.03 **(c)** 0.06 **(d)** 1 **Ans: (d)**

[Part – B]

SECTION- III [2 marks each]

19. Simplify : $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right), x \neq 0$ **Ans:** $\frac{1}{2}\tan^{-1}x$

20. If A is a non- singular square matrix of order 3 and $A^2 = 2A$, then find the value of $|A|$ **Ans:** $|A| = 8$

(OR)

If $X\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$, then find the matrix X **Ans:** $X = \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$

21. Determine the value of $'k'$ for which the following function is continuous at $x = 3$,**Ans:** $\boldsymbol{k =}$ **12** $f(x) = \begin{cases} \frac{(x+3)^2-36}{x-3} & , x \neq 3 \\ k & , x = 3 \end{cases}$

22. Find the equation of the normal to the curve $y = x + \frac{1}{x}, x > 0$ perpendicular to the line $3x - 4y = 7$ **Ans:** $\mathbf{8x + 6y - 31 = 0}$

23. Find $\int \frac{1}{\cos^2 x(1-\tan x)^2} dx$ **Ans:** $\boldsymbol{\frac{1}{1-\tan x} + c}$

(**OR**)

Find :$\int \frac{x\, dx}{1+ x \tan x}$ **Ans: log**$\boldsymbol{|\cos x + x \sin x| + c}$

24. Find the area of the region bounded by the parabola $y^2 = 8x$ and the line $x = 2$. **Ans**: $\mathbf{\frac{32}{3}}$ **sq. units**

25. Solve the following differential equation: $\frac{dy}{dx} = x^3 \operatorname{cosec} y$, given that $y(0) = 0$. **Ans**: $\mathbf{\cos y = 1 - \frac{x^4}{4}}$

26. Find the area of the parallelogram whose one side and a diagonal are represented by coinitial vectors
$\hat{\imath} - \hat{\jmath} + 2\hat{k}$ and $4\hat{\imath} + 5\hat{k}$ respectively. **Ans**: $\boldsymbol{\sqrt{42}}$ **sq. units**

27. The x- coordinate of a point on the line joining the points $P(2, 2, 1)$ and $Q(5, 1, -2)$ is 4. Find its z- coordinate. ? **Ans:** $\mathbf{-1}$

28. Probability of solving specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently, find the probability that the problem is solved. **Ans:**$\mathbf{\frac{2}{3}}$

(**OR**)

Given that E an F are events such that $P(E) = 0.8$, $(F) = 0.7$,$(E \cap F) = 0.6$. Find $P(\bar{E}/\bar{F})$ **Ans:**$\mathbf{\frac{1}{3}}$

SECTION- IV [3 marks each]

29. Check whether the relation **R** in the set **Z** of integers defined as R = {(a, b) : a + b is "divisible by 2"} is reflexive, symmetric or transitive.

30. If $x = 2\cos\theta - \cos 2\theta$, $y = 2\sin\theta - \sin 2\theta$, prove that $\frac{dy}{dx} = \tan\left(\frac{3\theta}{2}\right)$

31. Show that the function f given by $f(x) = |x - 3|$, $\forall\ x \in R$, is not differentiable at $x = 3$

(**OR**)

If $y = \tan\left(\frac{1}{a}\log y\right)$, show that $(1 + x^2)\frac{d^2y}{dx^2} + (2x - a)\frac{dy}{dx} = 0$

32. Find the intervals in which the function $f(x) = 4x^3 - 6x^2 - 72x + 30$ is

(a) strictly increasing (b) strictly decreasing. **Ans**: $\mathbf{(a)(-\infty, -2) \cup (3, \infty)(b)(-2,\ 3)}$

33. Evaluate :$\int \frac{2+\sin 2x}{1+\cos 2x} e^x dx$ **Ans:** $\boldsymbol{e^x \tan x + c}$

34. Find the area of the region in the first quadrant. bounded by x-axis, the line $x = \sqrt{3}y$ and the circle curves $x^2 + y^2 = 4$. **Ans:**$\boldsymbol{\frac{\pi}{3}}$

(**OR**)

Find the area of the ellipse $x^2 + 9y^2 = 36$ using integration. **Ans:**$\boldsymbol{12\pi}$ **sq. units**

35. Find the particular solution of the differential equation : $\log\left(\frac{dy}{dx}\right) = 3x + 4y$,given that $y = 0$ when $x = 0$ **Ans:** $\boldsymbol{4e^{3x} + 3e^{-4y} - 7 = 0}$

SECTION- V [5 marks each]

36. If A= $\begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, find $A^{-1}$and hence solve the given equations:

$x - 2y = 10,\ 2x - y - z = 8,\ -2y + z = 7$ **Ans:** $\boldsymbol{x = 0, y = -5, z = -3}$

(OR)

Find the product AB, where A = $\begin{bmatrix} \mathbf{1} & \mathbf{-1} & \mathbf{0} \\ \mathbf{2} & \mathbf{3} & \mathbf{4} \\ \mathbf{0} & \mathbf{1} & \mathbf{2} \end{bmatrix}$ **, B** = $\begin{bmatrix} \mathbf{2} & \mathbf{2} & \mathbf{-4} \\ \mathbf{-4} & \mathbf{2} & \mathbf{-4} \\ \mathbf{2} & \mathbf{-1} & \mathbf{5} \end{bmatrix}$ and use it to solve the equations: $x - y = 3, 2x + 3y + 4z = 17, y + 2z = 7$ **Ans:** $\boldsymbol{x = 2, y = -1, z = 4}$

37. Find the shortest distance between the lines:

$\vec{r} = (3\,\hat{\imath} + 2\hat{\jmath} - 4\,\hat{k}\,) + \lambda(\,\hat{\imath} + 2\hat{\jmath} + \,2\hat{k})$ and $\vec{r} = (5\,\hat{\imath} - 2\hat{\jmath}\,) + \mu(3\,\hat{\imath} + 2\hat{\jmath} + \,6\hat{k})$

If the lines intersect , find their point of intersection. **Ans:** $(\boldsymbol{-1, -6, -12})$

(OR)

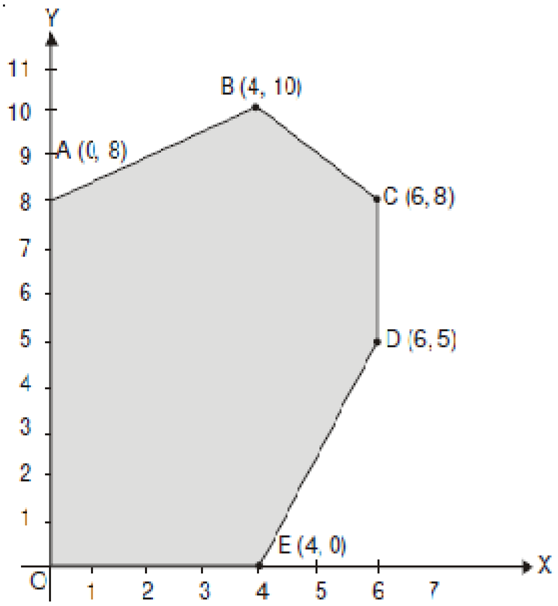
Find the foot of the perpendicular drawn from the point (−1, 3, −6) to the plane 2*x* + *y* − 2z + 5 = 0. Also find the equation and length of the perpendicular. **Ans:** $(\boldsymbol{-5,\ 1, -2}), \boldsymbol{\frac{x+1}{2} = \frac{y-3}{1} = \frac{z+6}{-2}, 6}$

38. Solve the following linear programming problem $(\mathrm{L.P.P.})$graphically:

Maximize Z= $x + 2y$ Subject to the constraints: $x + 2y \geq 100,\ \ 2\,x - y \leq 0,\ \ 2x + y \leq 200,$ $x \geq 0,\ \ y \geq 0$ **Ans: 250 at**$(\boldsymbol{50, 100})$

(OR)

The corner points of the feasible region determined by the system of linear constraints are as shown below:



Answer each of the following:

(i) Let $Z = 3x - 4y$ be the objective function. Find the maximum and minimum value of Z and also the corresponding points at which the maximum and minimum value occurs.
(ii) Let Z = $px + qy$, where $p, q > 0$ be the objective function. Find the condition on p and q so that the maximum value of Z occurs at B (4, 10) and C (6, 8). Also mention the number of optimal solutions in this case. **Ans: (i) Maximum 12 at$(4, 0)$; Minimum -32 at$(0, 8)$(ii) $p = q$, The number of optimal solutions are infinite.**

Kendriya Vidyalaya Sangathan, Raipur -Region

Time: 3 Hours **Sample Question Paper- II (2020 – 21)** M. M: 80

Class-XII

Subject: - Mathematics

General Instructions:--

1. This question paper contains two **Parts A** and **B**. Each part is compulsory. Part **A** carries **24** marks and Part **B** carries **56** marks.
2. **Part-A** has Objective Type Questions and **Part-B** has Descriptive Type Questions
3. Both Part A and Part B have choice.

Part-A

1. It consists of two sections- **I** and **II**
2. Section **I** comprises of **16** very short answer type questions.
3. Section **II** contains **2** case studies. Each case study comprises of 5 case –based MCQs. An examinee is to attempt **any 4 out of 5 MCQs.**

Part-B

1. It consists of three sections- **III, IV** and **V.**
2. Section **III** comprises of 10 questions of **2 marks** each.
3. Section **IV** comprises of 7 questions of **3 marks** each.
4. Section **V** comprises of 3 questions of **5 marks** each.
5. Internal choice is provided in **3** questions of section- III, **2** questions of section- IV and **3** questions of section- V. You have to attempt only one of the alternatives in all such questions.

[Part – A]

SECTION- I[1 mark each]

1. Show that the map $f : R \to R$ given by $f(x) = x^3 + 1$ is bijective.

(OR)

Check whether the relation R defined in the set $\{1,2,3,4,5,6\}$ as $R = \{(a, b): b = a + 1\}$ is reflexive, symmetric or transitive **Ans: None of these**

2. If A is a 3×3 invertible matrix, then what will be the value of k if $\det(A^{-1}) = (\det A)^k$

(OR)

If $A = \begin{bmatrix} 1 & 2 \\ -7 & 9 \end{bmatrix}$, then find adj A **Ans:** adj $A = \begin{bmatrix} 9 & -2 \\ 7 & 1 \end{bmatrix}$,

3. State the reason why the relation $R= \{(a, b) : a \le b^3\}$ on the set R of real number is not reflexive.?

4. Find the domain of the real function f defined by $f(x) = \sqrt{x-1}$ **Ans:** $\boldsymbol{D_f} = [\mathbf{1}, \infty), \boldsymbol{R_f} = [\mathbf{0}, \infty)$

5. Write the number of all possible matrices of order 2×3 with each entry 1 or 2 **Ans:$\mathbf{2^6}$**

6. Evaluate : $\int \sqrt{\sin 2x} \cos 2x \, dx$ **Ans:**$\boldsymbol{\frac{1}{3}(\sin 2x)^{\frac{3}{2}} + c}$

(OR)

Evaluate : $\int \log x \, dx$ **Ans:** $\boldsymbol{x.\log x - x + C}$

7. If $A = \begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}$ and $BA = (b_{ij})$, find $b_{21} + b_{32}$. **Ans: -18**

8. Evaluate : $\int_0^1 5^x\,dx$ **Ans:** $\mathbf{\frac{1}{\log 5}}$

9. If $\sin x$ is an integrating factor of differential equation $\frac{dy}{dx} + \mathrm{P}y = \mathrm{Q}$, then find P **Ans:** $\boldsymbol{P = \cot x}$

(OR)

Find the sum of the degree and the order for the following differential equation : $\frac{d}{dx}\left[\left(\frac{d^2y}{dx^2}\right)^4\right] = 0$ **Ans: order 3, degree1 Sum=4**

10. Write the value of $\hat{\imath}.(\hat{\jmath}\times\hat{k}) + \hat{\jmath}.(\hat{k}\times\hat{\imath}) + \hat{k}.(\hat{\imath}\times\hat{\jmath})$. **Ans: 3**

(OR)

Find the projection of $\vec{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$ on $\vec{b} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$ **Ans:** $\mathbf{\frac{5}{6}\sqrt{6}}$

11. If $\vec{a}$ and $\vec{b}$ are two non-zero vectors such that $|\vec{a}\times\vec{b}| = |\vec{a}.\vec{b}|$ then find the angle between $\vec{a}$ and $\vec{b}$ **Ans:** $\boldsymbol{\frac{\pi}{4}}$

12. If $\hat{\jmath} + \hat{k}$ and $3\hat{\imath} - \hat{\jmath} + 4\hat{k}$ represent the two sides AB and AC, respectively of Δ ABC. Find the length of median through A **Ans:** $\mathbf{\frac{\sqrt{34}}{2}}$

13. Find the vector equation of the plane which is at a distance of 7 units from the origin and its normal vector $2\,\hat{\imath} - 3\hat{\jmath} + 6\hat{k}$ **Ans:** $\boldsymbol{\vec{r}.\frac{1}{7}(2\,\hat{\imath} - 3\hat{\jmath} + 6\hat{k}) = 7}$

14. Find, $P(A \cap B)$; if $2P(A) = P(B) = \frac{5}{13}$ and $P(A/B) = \frac{2}{5}$ **Ans:** $\mathbf{\frac{2}{13}}$

15. Let A and B be events with $P(A) = \frac{3}{5}$, $P(B) = \frac{5}{10}$ and $P(A \cap B) = \frac{1}{5}$. Are A and B independent events?

16. Find the distance between the given planes: $2x + y + 2z - 8 = 0$ and $4x + 2y + 4z + 5 = 0$ **Ans:** $\mathbf{\frac{7}{2}}$

SECTION- II For Q.17 and Q18 attempt any four (MCQs) [1 mark each]

17. An insurance company insures three type of vehicles i.e., type A, B and C. If it insured 12000 vehicles of type A, 16000 vehicles of type B and 20,000 vehicles of type C. Survey report says that the chances of their accident are 0.01, 0.03 and 0.04 respectively.

(**Based on the informations given above, write the answer of following**)

(i) The probability of insured vehicle of type C is

(a) $\frac{5}{12}$ **(b)** $\frac{4}{12}$ **(c)** $\frac{7}{12}$ **(d)** $\frac{3}{12}$ **Ans: (a)**

(ii) Let E be the event that insured vehicle meets with an accident, then $P(E/A)$ is **Ans: (b)**

(a) 0.09 **(b) 0.01** **(c) 0.07** **(d) 0.06**

(iii) Let E be the event that insured vehicle meets with an accident, then $P(E)$ is **Ans: (d)**

(a) $\frac{38}{1200}$ (b) $\frac{32}{1200}$ (c) $\frac{24}{1200}$ (d) $\frac{34}{1200}$

(iv) The probability of an accident that one of the insured vehicle meets with an accident and it is a type C vehicle is **(a)** $\frac{2}{7}$ **(b)** $\frac{3}{7}$ **(c)** $\frac{5}{7}$ **(d)** $\frac{4}{7}$ **Ans: (d)**

(v) One of the insured vehicles meets with an accident and it is not of type A and C is **Ans: (a)**

(a) $\frac{12}{35}$ **(b)** $\frac{20}{35}$ **(c)** $\frac{1}{35}$ **(d)** $\frac{17}{35}$

18. Mr. Gaurav is a student of class XII of a school, his mathematics teacher Mrs. Swati Sharma after completing the topic maxima and minima gave him the function $f(x) = sin\, 2x + x, \ -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ and asked the following questions to

(i) The point at which the function attains local maxima or local minima

(a) $\boldsymbol{x = \pm\frac{\pi}{3}}$ **(b)** $\boldsymbol{x = \pm\frac{\pi}{4}}$ **(c)** $\boldsymbol{x = \pm\frac{\pi}{6}}$ **(d)** $\boldsymbol{x = 0}$ **Ans: (c)**

(ii) The local maximum value of $f(x)$ is

(a) $\left(\frac{1}{\sqrt{2}} - \frac{\pi}{4}\right)$ **(b)** $\left(\frac{\sqrt{3}}{2} - \frac{\pi}{6}\right)$ **(c)** $\left(\frac{\pi}{6} + \frac{\sqrt{3}}{2}\right)$ **(d)** $\left(\frac{\pi}{4} + \frac{1}{\sqrt{2}}\right)$ **Ans: (b)**

(iii) The local minimum value of $f(x)$ is

(a) $\left(\frac{\sqrt{3}}{2} - \frac{\pi}{6}\right)$ **(b)** $\left(-\frac{\sqrt{3}}{2} + \frac{\pi}{4}\right)$ **(c)** $\left(\frac{\pi}{6} - \frac{\sqrt{3}}{2}\right)$ **(d)** $\left(\frac{\sqrt{3}}{2} - \frac{\pi}{4}\right)$ **Ans: (c)**

(iv) The value of $f''(x)$ at $x = \frac{\pi}{4}$ is

(a) -4 **(b)** 1 **(c)** -1 **(d)** 4 **Ans: (a)**

(v) Function attains the local minimum value at the point when x is equal to

(a) $\boldsymbol{x = \frac{\pi}{6}}$ **(b)** $\boldsymbol{x = -\frac{\pi}{6}}$ **(c)** $\boldsymbol{x = \frac{\pi}{3}}$ **(d)** $\boldsymbol{x = -\frac{\pi}{3}}$ **Ans: (b)**

[Part – B]

SECTION- III [2 marks each]

19. Find the value of $\tan^2(\sec^{-1}2) + \cot^2(\text{cosec}^{-1}3)$ **Ans: 11**

20. If A is a skew – symmetric matrix of order 3, then prove that det A= 0

(OR)

If A is a square matrix such that $A^2 = I$, then write the value of $(A - I)^3 + (A + I)^3 - 7A$, where I is an identity matrix. **Ans: A**

21. If $|\vec{a} + \vec{b}| = 60$, $|\vec{a} - \vec{b}| = 40$ and $|\vec{a}| = 22$, then find $|\vec{b}|$ **Ans: 46**

22. Evaluate: $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos x}{1+e^x} dx$ **Ans: 1**

(OR)

Find the value of $\int \tan^{-1}\left(\frac{\sin 2x}{1+\cos 2x}\right)dx$. **Ans:** $\boldsymbol{\frac{x^2}{2}+C}$

23. Find the value of k, for which f $(x) = \begin{cases} \frac{\sqrt{1+kx}-\sqrt{1-kx}}{x}, \text{if} -1 \le x < 0 \\ \frac{2x+1}{x-1}, \quad \text{if } 0 \le x < 1 \end{cases}$ is continuous at $x = 0$

Ans: $\mathbf{k = -1}$

24. Find a point on the curve $y = (x-3)^2$, where the tangent is parallel to the line joining the points A$(4, 1)$ and B$(3, 0)$. **Ans:** $\boldsymbol{\left(\frac{7}{2}, \frac{1}{4}\right)}$

25. Find the area bounded by the curve $y = 3x$, x- axis and between the ordinates $x = 1$ and $x = 3$ **Ans: 12 sq.units**

26. Solve the differential equation:$(1 + e^{2x})dy \ (1 + y^2)dx = 0$ given that $x = 0, y = 1$ **Ans:** $\boldsymbol{\tan^{-1}y + \tan^{-1}(e^x) = \frac{\pi}{2}}$

27. The equation of a line is given by $\frac{2x-5}{4} = \frac{y+4}{3} = \frac{6-z}{6}$. Write the direction cosines of a line parallel to given line. **Ans:**$\boldsymbol{< \pm\frac{2}{7}, \pm\frac{3}{7}, \pm\frac{6}{7} >}$

(OR)

Find the coordinates of the point, where the line $\frac{x+1}{2} = \frac{y+2}{3} = \frac{z+3}{4}$ intersects the plane $x + y + 4z = 6$. **Ans:**$\boldsymbol{(1, 1, 1)}$

28. A bag contains 23 tickets, numbered from 1 to 23. Two tickets are drawn one by one without replacement. Find the probability that both the tickets will show the even numbers. **Ans:** $\boldsymbol{\frac{5}{23}}$

SECTION- IV [3 marks each]

29. Let A= $\{x \in Z: 0 \le x \le 12\}$ .Show that R $= \{(a, \mathrm{b}): a, \mathrm{b} \in \mathrm{A}, |a - \mathrm{b}| \text{ is divisible by } 4\}$ is an equivalence relation

30. If $x = a\left(\cos\theta + \log\tan\frac{\theta}{2}\right)$ and $y = a\sin\theta$, find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$ **Ans:**$\boldsymbol{\frac{2\sqrt{2}}{a}}$

(OR)

If $(\cos x)^y = (\sin y)^x$, find $\frac{dy}{dx}$ **Ans:** $\boldsymbol{\frac{\log\sin y + y\tan x}{\log\cos x - x\cot y}}$

31. If $y =$ A $\cos(\log x)$ +B $\sin(\log x)$, prove that :$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

32. Separate the interval $\left[0, \frac{\pi}{2}\right]$ into sub intervals in which the function $f(x) = sin^4x + cos^4x$ is (i) increasing (ii) decreasing **Ans: decreasing in $\boldsymbol{\left[0, \frac{\pi}{4}\right)}$, increasing in$\boldsymbol{(\frac{\pi}{4}, \frac{\pi}{2}]}$**

33. Find the particular solution of the differential equation: $\frac{dy}{dx} = -\frac{x+y\cos x}{1+\sin x}$ given that $y =$ 1when $x = 0$ **Ans**: $\boldsymbol{y = -\frac{1}{2}x^2\frac{1}{(1+sin\,x)} + \frac{1}{(1+sin\,x)}}$

(OR)

Solve the differential equation: $x\cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y\cos\left(\frac{y}{x}\right) + x$. **Ans: sin** $\boldsymbol{\left(\frac{y}{x}\right) = \log|Cx|}$

34. Evaluate:$\int\left(\sqrt{\cot x} + \sqrt{\tan x}\right)dx$ **Ans:**$\boldsymbol{\sqrt{2}tan^{-1}\left(\frac{tan\,x-1}{\sqrt{2tan\,x}}\right) + c}$

35. Find the area bounded by the curve $y = sin\,x$ between $x = 0$ and $x = 2\pi$ **Ans: 4 sq.units**

SECTION- V [5 marks each]

36. Solve the following L.P.P. graphically: Maximise $Z = 4x + y$, Subject to following constraints

$x + y \leq 50,\ \ 3x + y \leq 90,\ \ x \geq 10,\ \ x \geq 0, y \geq 0$ **Ans**: **120 at(30, 0)**

(OR)

Solve the following L.P.P. graphically: Maximise $Z = 20x + 10y$, Subject to following constraints

$x + 2y \leq 28,\ \ 3x + y \leq 24,\ \ x \geq 2,\ \ x \geq 0, y \geq 0$ **Ans**: **200 at(4, 12)**

37. If A= $\begin{bmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{bmatrix}$, find $A^{-1}$and hence solve the given equations:

$2x + y + 3z = 3,\ 4x - y = 3,\ -7x + 2y + z = 2$ **Ans:** $\boldsymbol{x = -6, y = -27, z = 14}$

(OR)

Solve the system of equations: $x - y + z = 4,\ x - 2y - 2z = 9, 2x + y + 3z = 1$

Ans: $\boldsymbol{x = 3, y = -2, z = -1}$

38. Show that the four points $(0, -1, -1)$,$(4, 5, 1)$, $(3, 9, 4)$ and $(-4, 4, 4)$ are coplanar. Also find the equation of the plane containing them. **Ans:**$\boldsymbol{5x - 7y + 11z + 4 = 0}$

(OR)

Find the shortest distance between the following pair of lines :

$\frac{x-6}{1} = \frac{y-2}{-2} = \frac{z-2}{2};\quad \frac{x+4}{3} = \frac{y}{-2} = \frac{z+1}{-2}.$ **Ans: 9**

Kendriya Vidyalaya Sangathan Raipur -Region

Time: 3 Hours **Sample Question Paper- III (2020 – 21)** M. M: 80

Class-XII

Subject: - Mathematics

General Instructions:--

1. This question paper contains two **Parts A** and **B**. Each part is compulsory. Part **A** carries **24** marks and Part **B** carries **56** marks.
2. **Part-A** has Objective Type Questions and **Part-B** has Descriptive Type Questions
3. Both Part A and Part B have choice.

Part-A

1. It consists of two sections- **I** and **II**
2. Section **I** comprises of **16** very short answer type questions.
3. Section **II** contains **2** case studies. Each case study comprises of 5 case –based MCQs. An examinee is to attempt **any 4 out of 5 MCQs.**

Part-B

1. It consists of three sections- **III, IV** and **V.**
2. Section **III** comprises of 10 questions of **2 marks** each.
3. Section **IV** comprises of 7 questions of **3 marks** each.
4. Section **V** comprises of 3 questions of **5 marks** each.
5. Internal choice is provided in **3** questions of section- III, **2** questions of section- IV and **3** questions of section- V. You have to attempt only one of the alternatives in all such questions.

[Part – A]

SECTION- I[1 mark each]

1. A die, whose faces are marked 1, 2, 3 in red and 4, 5, 6 in green, is tossed. Let A be the event " number obtained is even" and B be the event "number obtained is red " . Find if A and B are independent events. **Ans: A and B are not independent**

2. If $\vec{a}$, and $\vec{b}$ are two unit vectors inclined to x- axis at angles 45^0 and 135^0 respectively, then find the value of $|\vec{a}+\vec{b}|$ **Ans:** $\sqrt{\mathbf{2}}$

3. Find the value of $a+b$, if the point $(2,\ a,\ 3)$, $(3,-5,\ b)$ and $(-1,\ 11,\ 9)$ are collinear.
Ans: $\boldsymbol{a=-1, b=1, a+b=0}$

4. Find $|\vec{x}|$, if for a unit vector $\vec{a}$, $(\vec{x}-\vec{a}).(\vec{x}+\vec{a})=15$ **Ans: 4**

(OR)

Find a unit vector in the direction of $(\vec{a}+\vec{b})$ where $\vec{a}=-\hat{\imath}+\hat{\jmath}+\hat{k}$ and $\vec{b}=2\hat{\imath}+\hat{\jmath}-3\hat{k}$
Ans: $\frac{1}{3}(\hat{\imath}+2\hat{\jmath}-2\hat{k})$

5. If A and B are square matrices of order 3 each, $|A|=2$ and $|B|=3$ .Find the value of $|3AB|$
Ans: 162

(OR)

If the matrix $A=\begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$ is skew symmetric, find the values of 'a' and 'b'.

Ans: $\boldsymbol{a=-2, b=3}$

6. A random variable X has the following probability distribution: Determine k

Ans: $\frac{1}{10}$

X	0	1	2	3	4	5	6	7
P (X)	0	2k	3 k	k	2k	k^2	$7\,k^2$	$2\,k^2+k$

7. For a 3×3 matrix A, given that $|A|=7$, find $|adj\ A|$ **Ans: 49**

8. If $\begin{bmatrix} x+4 & 3y \\ 8 & -6 \end{bmatrix}=\begin{bmatrix} 2x+2 & y+2 \\ 0 & x-8y \end{bmatrix}$, then find the value of $x-3y$ **Ans:** $\boldsymbol{-1}$

9. Find the sum of the degree and the order for the following differential equation :

$\left(\frac{d^2y}{dx^2}\right)+\sqrt[3]{\frac{dy}{dx}}+(5+x)=0$ **Ans: order 2, degree 3 Sum=5**

Solve that the differential equation **:** $\frac{dy}{dx}=\frac{1-\cos x}{1+\cos x}$ **Ans:** $\boldsymbol{y=2\tan\frac{x}{2}-x+C}$

10. Find : $\int\frac{dx}{\sqrt{9+8X-x^2}}$ **Ans:** $\boldsymbol{\sin^{-1}\left(\frac{x-4}{5}\right)+c}$

(OR)

Evaluate : $\int_0^1\frac{\tan^{-1}x}{1+x^2}\,dx$ **Ans:** $\boldsymbol{\frac{\pi^2}{32}}$

11. Evaluate : $\int_e^{e^2}\frac{1}{x.\log x}\,dx$ **Ans:** $\boldsymbol{\log 2}$

12. If α,β,γ are the angles which a given line makes with positive direction of the axes, then prove that $\sin^2\alpha+\sin^2\beta+\sin^2\gamma=2$

13. Find the vector equation of the line joining the points whose position vectors are $2\,\hat{\imath}-\hat{\jmath}+\hat{k}$ and $\hat{\imath}+2\,\hat{\jmath}-3\,\hat{k}$ **Ans:** $\vec{r}=2\,\hat{\imath}-\hat{\jmath}+\hat{k}+\boldsymbol{\lambda(-\hat{\imath}+3\hat{\jmath}-\hat{k})}$

14. Let $A=\{1,2,3,4\}$ and R be the equivalence relation in A× A defined by $(a, b)\,^R(c, d)$ iff $a+d=b+c$ for all a, b, c ,d $\in$ A Find the equivalence class [(1,3] **Ans:** $\boldsymbol{[(1,3]=\{(1,3),(2,4)\}}$

15. Let $R\{(a,a),(b,b),(c,c),(a,b)\}$ on the set $A=\{a,b,c\}$, then write, which type of relation is it ? **Ans: reflexive**

16. Let $f:R\to R$ be defined as $f(x)=10\,x+7$. Show that f is both injective and surjetive

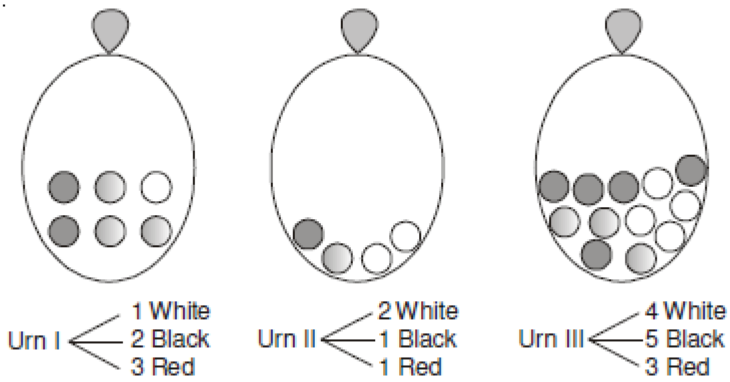
(OR)

Let f, g : R → R be defined as $f(x)=|x|$ and $g(x)=[x]$, then find the value of $f\left(-\frac{5}{2}\right)$ and $g\left(-\sqrt{2}\right)$ **Ans:** $\boldsymbol{\frac{5}{2}, -2}$

SECTION- II For Q.17 and Q18 attempt any four (MCQs) [1 mark each]

17. There are three Urn having different coloured balls. The contents of Urns I, II, III are as follows:

Urn I : 1 white, 2 black and 3 red balls
Urn II: 2 white, 1 black and 1 red ball
Urn III: 4 white, 5 black and 3 red balls



Based on the above information answer the following questions:

(i) The probability that one white and one red ball is drawn only from Urn I is
(b) $\frac{1}{5}$ **(b)** $\frac{1}{2}$ **(c)** $\frac{1}{3}$ **(d)** $\frac{2}{3}$ **Ans: (c)**

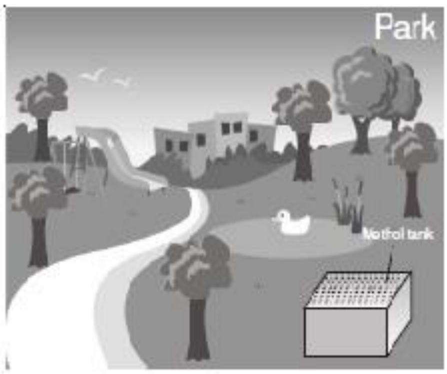
(ii) The probability of selecting any one of the urn is
(b) 1 **(b)** $\frac{2}{3}$ **(c)** $\frac{1}{2}$ **(d)** $\frac{1}{3}$ **Ans: (d)**

(iii) Using Baye's Theorem the probability that balls are drawn from Urn I is **Ans: (c)**
(b) $\frac{55}{118}$ **(b)** $\frac{65}{118}$ **(c)** $\frac{33}{118}$ **(d)** $\frac{36}{118}$

(iv) The total probability of getting 1 white and 1 red ball is **Ans: (a)**
(a) $\frac{118}{495}$ **(b)** $\frac{103}{165}$ **(c)** $\frac{103}{495}$ **(d)** $\frac{68}{165}$

(v) Probability that the balls are not drawn from III Urn is **Ans: (b)**
(a) $\frac{15}{59}$ **(b)** $\frac{44}{59}$ **(c)** $\frac{17}{59}$ **(d)** $\frac{27}{59}$

18. In a park, an open tank is to be constructed using metal sheet with a square base and vertical sides so that it contains 500 cubic meter of water.



Based on the above information answer the following questions:

(i) If the edge of square is *x* meter and height of tank is *y* m then correct relation is
(a) $x^2y = 500$ **(b)** $xy = 500$ **(c)** $x^2y^2 = 500$ **(d)** $x^2 + y = 500$ **Ans: (a)**

(ii) Relation for surface area of tank in terms of *x* and *y* is
(a) $2x^2 + 4xy$ **(b)** $x^2 + xy$ **(c)** $2x^2 + 2xy$ **(d)** $x^2 + 4xy$ **Ans: (d)**

(iii) The surface area of tank is minimum when *x* is equal to
(a) 8 m **(b)** 10 m **(c)** 20 m **(d)** 5 m **Ans: (b)**

(iv) The minimum surface area of tank is
(a) $200\ sq.m$ **(b)** $300\ sq.m$ **(c)** $250\ sq.m$ **(d)** $400\ sq.m$ **Ans: (b)**

(v) If size of square base of tank become twice and height remains same, then the volume of tank will increase by cubic meters *i.e.*,

(a) $500\ cu\ m$ **(b)** $1000\ cu\ .m$ **(c)** $1500\ cu\ .m$ **(d)** $2000\ cu.m$

Ans: (c)

[Part – B]

SECTION- III [2 marks each]

19. Find the value of : $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)+\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$. **Ans:** $-\frac{\pi}{12}$

20. Find the value of 'a' for which the function f defined as : $f(x)=\begin{cases} a\sin\frac{\pi}{2}(x+1), & x\le 0\\ \frac{\tan x-\sin x}{x^3}, & x>0\end{cases}$ **is continuous at $x=0$** **Ans:** $a=\frac{1}{2}$,

21. If A= $\begin{bmatrix}0 & 3\\ 2 & -5\end{bmatrix}$ and KA= $\begin{bmatrix}0 & 4a\\ -8 & 5b\end{bmatrix}$, find the values of K and a . **Ans:** $K=-4, a=-3$

(OR)

Find the value k, if the area of triangle formed by vertices A(k, 0), B(5, 2) and C(1, 4) is 5 sq. units. **Ans:** $K=4\ 0r\ 14$

22. It is given that at $x=1$ the function x^4-62x^2+ax+9 attains its maximum value, on the interval $[0,2]$, find the value of "a" **Ans:** $a=120$

23. Evaluate: $\int_0^{\frac{\pi}{2}}\frac{\tan^7 x}{\cot^7 x+\tan^7 x}dx$ **Ans:** $\frac{\pi}{4}$

(OR)

Find : $\int x\,.\tan^{-1}x\,dx$. **Ans:** $\frac{x^2}{2}\tan^{-1}x-x+\frac{1}{2}\tan^{-1}x+C$

24. The area between $x=y^2$ and $x=4$ is divided into two equal parts by the line $x=a$. find the value of "a" **Ans:** $a=(4)^{\frac{2}{3}}$

25. Let If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three vectors such that $|\vec{a}|=3$, $|\vec{b}|=4$ and $|\vec{c}|=5$ and each one of them is perpendicular to the sum of other two, find $|\vec{a}+\vec{b}+\vec{c}|$ **Ans:** $5\sqrt{2}$

26. Find the particular solution of the differential equation: $\frac{dy}{dx}=-4xy^2$ given that $y=1$when $x=0$ **Ans:**$y=\frac{1}{2x^2+1}$

27. Show that the lines $\frac{x-1}{3}=\frac{y-1}{-1}$, $z+1=0$ and $\frac{x-4}{2}=\frac{z+1}{3}$, $y=0$ intersect each other. Also find their point of intersection. **Ans:**$(4,0,-1)$

28. A family has two children. What is the probability that both the children are boys given that at least one of them is a boy? **Ans:**$\frac{1}{3}$

(OR)

If A and B are independent events such that P(A) = 0.3, P(B) = 0.6, then find P(neither A nor B)

Ans: 0.28

SECTION- IV [3 marks each]

29. Consider a function f : R+→ $[15,\infty)$ given by $f(x)=4x^2+12x+15$. Show that f is bijective function.

30. If $x^y=e^{x-y}$, then show that $\frac{dy}{dx}=\frac{\log x}{\{\log(xe)\}^2}$

(OR)

If $\cos y = x\cos(a+y)$, with$\cos a \neq \pm 1$, prove that $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$

31. If $x = a\sin t$, and $y = \log\tan\left(\frac{\pi}{4}+\frac{t}{2}\right)$, Find $\frac{dy}{dx}$ when t = $\frac{\pi}{4}$ **Ans**: $\boldsymbol{\frac{2}{a}}$

32. Evaluate:$\int_0^{\frac{\pi}{2}}(2\log\sin x - \log\sin 2x)\,dx$ **Ans:** $\boldsymbol{-\frac{\pi}{2}\ log 2}$

(OR)

Evaluate:$\int\frac{dx}{x(x^3+8)}$ **Ans:**$\boldsymbol{\frac{1}{24}\ log\left|\frac{x^3}{x^3+8}\right| + c}$

33. Find the general solution of the differential equation : $x\,dy - (y+2x^2)dx = 0$ **Ans:** $\boldsymbol{y = 2x^2 + cx}$

34. Find the equation of the normal to the curve $x^2 = 4y$, which passes through the point $(1,2)$ **Ans:** $\boldsymbol{x + y = 3}$

35. Find the area of the region bounded by the curve $y^2 = 4x$, y-axis, and the line $y = 3$ **Ans:**$\boldsymbol{\frac{9}{4}\ sq\ units}$

SECTION- V [5 marks each]

36. Find the vector equation of the plane passing through the intersection of the planes $\vec{r}.\left(2\,\hat{\imath}+\hat{\jmath}+3\,\hat{k}\right) = 7$ and $\vec{r}.\left(2\hat{\imath}+5\,\hat{\jmath}+3\,\hat{k}\right) = -5$ and the point (2,1,3).**Ans:**$\boldsymbol{\vec{r}.\left(58\,\hat{\imath} - 3\hat{\jmath} + 12\hat{k}\right) = 49}$

(OR)

What is the distance of the point $(-1,-5,-10)$from the point of intersection of the line$\vec{r} = 2\hat{\imath}-\hat{\jmath}+2\hat{k}+\boldsymbol{\lambda}\left(3\hat{\imath}+4\hat{\jmath}+2\hat{k}\right)$ and the plane $\vec{r}.\left(\hat{\imath}-\hat{\jmath}+\hat{k}\right) = 5$ **Ans:13**

37. Solve the following linear programming problem (L. P. P.)graphically:
Maximize Z= $30x + 60y$ Subject to the constraints: $2x + y \leq 70$, $x + y \leq 40$, $x + 3y \leq 90$, $x \geq 0$, $y \geq 0$ **Ans: 1950 at$(\boldsymbol{15, 25})$**

(OR)

Solve the following linear programming problem (L. P. P.)graphically:
Minimize Z= $5x + 10y$ Subject to the constraints: $x + 2y \leq 120$, $x + y \geq 60$, $x - 2y \geq 0$, $x \geq 0$, $y \geq 0$ **Ans: 300 at$(\boldsymbol{60, 0})$**

38. Find the values of x , y and z, if $\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfies $\mathrm{A}' = \mathrm{A}^{-1}$ **Ans:** $\boldsymbol{x = \pm\frac{1}{\sqrt{2}}, y = \pm\frac{1}{\sqrt{6}}, x = \pm\frac{1}{\sqrt{3}}}$

(OR)

Verify : $\mathrm{A}(\mathrm{adj}\ \mathrm{A}) = (\mathrm{adj}\ \mathrm{A})\mathrm{A} = |\mathrm{A}|\mathrm{I}$ for matrix A= $\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$ **Ans:** $|\mathbf{A}| =$ **11 and verify**

Kendriya Vidyalaya Sangathan Raipur -Region

Time: 3 Hours **Sample Question Paper- IV (2020 – 21)** M. M: 80

Class-XII

Subject: - Mathematics

General Instructions:--

1. This question paper contains two **Parts A** and **B**. Each part is compulsory. Part **A** carries **24** marks and Part **B** carries **56** marks.
2. **Part-A** has Objective Type Questions and **Part-B** has Descriptive Type Questions
3. Both Part A and Part B have choice.

Part-A

1. It consists of two sections- **I** and **II**
2. Section **I** comprises of **16** very short answer type questions.
3. Section **II** contains **2** case studies. Each case study comprises of 5 case –based MCQs. An examinee is to attempt **any 4 out of 5 MCQs.**

Part-B

1. It consists of three sections- **III, IV** and **V.**
2. Section **III** comprises of 10 questions of **2 marks** each.
3. Section **IV** comprises of 7 questions of **3 marks** each.
4. Section **V** comprises of 3 questions of **5 marks** each.
5. Internal choice is provided in **3** questions of section- III, **2** questions of section- IV and **3** questions of section- V. You have to attempt only one of the alternatives in all such questions.

[Part – A]

SECTION- I[1 mark each]

1. If R = {(x, y) : $x + 3y = 9$} is a relation on N. Write the range of R. **Ans:** $\{\mathbf{1},\mathbf{2}\}$

2. Using the principal value, evaluate the following: $\cos^{-1}\left(\cos\frac{2\pi}{3}\right)+\sin^{-1}\left(\sin\left(-\frac{\pi}{6}\right)\right)+\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$

   Ans: $\boldsymbol{\frac{\pi}{4}}$

   (OR)

   Evaluate: $\sin\left[\frac{\pi}{6}-\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]$ **Ans: 1**

3. Write the value of $2x-3y+z$ if $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix}=\begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$ **Ans: 4**

4. If $3\tan^{-1}x+\cot^{-1}x=\pi$, then find x **Ans: 1**

5. If A = $\begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$, then show that $A^{-1}=\frac{1}{19}A$

6. For what value of x, the matrix :- $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$ is singular? **Ans: 3**

   (OR)

   If A is a square matrix of order 3 and $|3A|=K|A|$, then write the value of K **Ans: 27**

7. Prove that if E and F are independent events, then the events E and F′ are also independent.

8. A and B throw a pair of dice alternatively. A wins the game if he gets a total 7 and B wins the game if he gets a total 10. Write their favorable cases. **Ans: A,total 6; B total 3 favorable cases.**

9. Evaluate : $\int (3x-4)^3 dx$ **Ans:**$\boldsymbol{\frac{1}{12}(3x-4)^4+c}$

10. Evaluate : $\int_0^5 (x+1)\,dx$ **Ans:**$\boldsymbol{\frac{35}{2}}$

(**OR**)

Evaluate**:** $\int_{-2}^{1} \frac{|x|}{x}\,dx$ **Ans:**$\boldsymbol{-1}$

11. Find the order and degree of the differential equation: $\log\left(\frac{dy}{dx}\right)+\frac{d^3y}{dx^3}=y$**Ans:Order 3, degree not def.**

(**OR**)

Find the integrating factor of differential equation:$(x^2+1)\frac{dy}{dx}+2xy=\sqrt{x^2+4}$ **Ans:I. F.** $=$ $\boldsymbol{(x^2+1)}$

12. Find the value of ‘p’ for which vectors $3\hat{\imath}+2\hat{\jmath}+9\hat{k}$ and $\hat{\imath}-2p\hat{\jmath}+3\hat{k}$ are parallel.**Ans:**$\boldsymbol{-\frac{1}{3}}$

(**OR**)

Write the coordinates of the point which is the reflection of the point $(\alpha,\ \beta,\ \gamma)$ in the XZ – plane.

Ans: $\boldsymbol{(\alpha,-\beta,\ \gamma)}$

13. Find the position vector of the point which divides the join of the points with position vectors $\vec{a}+3\vec{b}$ and $\vec{a}-\vec{b}$ internally in the ratio $1:3$. **Ans:** $\vec{a}+2\vec{b}$

14. Let $\vec{b}=\hat{\imath}+2\hat{\jmath}-2\hat{k}$ and $\vec{a}=8\hat{\imath}+\hat{\jmath}$, find the projection of $\vec{a}$ on $\vec{b}$ **Ans:**$\boldsymbol{\frac{10}{3}}$

15. Find the equation of the plane passing through the point$(2,\ 1,-3)$ that is parallel to the plane $\vec{r}.(\hat{\imath}-2\hat{\jmath})=7$ **Ans:** $\boldsymbol{x-2y=0}$

16. Find the equation of the line joining the points $P(2,\ 5,-7)$ and $Q(-1,\ 3,4)$ in vector form.

$$\boldsymbol{\vec{r}=(2\hat{\imath}+5\hat{\jmath}-7\hat{k})+\lambda(-3\hat{\imath}-2\hat{\jmath}+11\hat{k})}$$

SECTION- II For Q.17 and Q18 attempt any four (MCQs) [1 mark each]

17. Ms. Manisha and Ms. Ritu are two friends. Ms. Manisha has 4 black and 6 red balls in her bag, where as Ms. Ritu has 7 black and 3 Red balls in her bag. They decided to throw a die and to draw the balls from their bags in such away that, if 1 or 2 appears on die then ball will be drawn from Ms. Manisha’s bag otherwise balls will be drawn from Ms. Ritu’s bag.

On the basis of this situation answer the followings:

(i) The probability that Ms. Ritu's bag is not selected is

(a) 0 **(b)** $\frac{2}{3}$ **(c)** $\frac{1}{3}$ **(d)** $\frac{1}{2}$ **Ans:** $(\boldsymbol{c})$

(ii) The probability that Ms. Manisha's bag is selected

(a) $\frac{1}{2}$ **(b)** 1 **(c)** $\frac{2}{3}$ **(d)** $\frac{1}{3}$ **Ans:** $(\boldsymbol{d})$

(iii) Probability if two balls are drawn at random (without replacement) in which 1 is red and 1 is black and drawn from Ms. Ritu's bag is

(a) $\frac{24}{45}$ **(b)** $\frac{21}{45}$ **(c)** $\frac{14}{45}$ **(d)** $\frac{19}{45}$ **Ans:** $(\boldsymbol{b})$

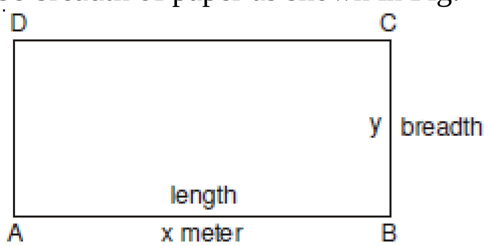
(iv) Probability if two balls are drawn at random (without replacement) in which 1 is red and other black and drawn from Ms. Manisha's is

(a) $\frac{7}{23}$ **(b)** $\frac{21}{45}$ **(c)** $\frac{24}{45}$ **(d)** $\frac{19}{23}$ **Ans:** $(\boldsymbol{c})$

(v) The total probability of drawing 1 red and 1 black ball is

(a) $\frac{22}{45}$ **(b)** $\frac{23}{45}$ **(c)** $\frac{17}{23}$ **(d)** $\frac{15}{23}$ **Ans:** $(\boldsymbol{a})$

18. An artist wishes to purchase a coloured sheet of paper of perimeter 100 cm. Let *x* be the length and *y* be breadth of paper as shown in Fig.



On the basis of this situation answer the followings:

(i) The correct relation between *x* and *y* is

(a) $y = 50 + x$ **(b)** $\boldsymbol{x} + \boldsymbol{y} = 100$ **(c)** $y = 50 - x$ **(d)** $\boldsymbol{xy} = 100$ **Ans:** $(\boldsymbol{c})$

(ii) The area *A* of sheet *ABCD* as a function of *x* is

(a) $50x + x^2$ **(b)** $\frac{1}{2}(50x - x^2)$ **(c)** $50x - x^2$ **(d)** $\frac{1}{2}(50x + x^2)$ **Ans:** $(\boldsymbol{c})$

(iii) The area ABCD of paper be maximum when *x* is equal to

(a) 50 cm **(b)** 35 cm **(c)** 20 cm **(d)** 25 cm **Ans:** $(\boldsymbol{d})$

(iv) Maximum area of paper is equal to

(a) 400 sq. cm **(b)** 2500 sq. cm **(c)** 625 sq. cm **(d)** 900 sq. cm **Ans:** $(\boldsymbol{c})$

(v) If a circle is draw through its vertices then the radius of circle is

(a) $\frac{20}{\sqrt{2}}$ cm **(b)** $\frac{25}{\sqrt{2}}$ cm **(c)** $\frac{50}{\sqrt{2}}$ cm **(d)** $\frac{35}{\sqrt{2}}$ cm **Ans:** $(\boldsymbol{b})$

[Part – B]

SECTION- III [2 marks each]

19. A card is drawn from a will shuffled deck of 52 cards. The outcome is noted, the card is replaced and the deck reshuffled. Another card is drawn from the deck. What is the probability that the first card is an ace and the second card is a red queen. **Ans:** $\frac{\mathbf{1}}{\mathbf{338}}$

(OR)

10 % of the bulbs produced in a factory are red colour and 2% are red and defective. If one bulb is picked at random , determine the probability of its being defective if it is red ? **Ans**: $\frac{1}{5}$

20. Write the sum of the intercepts cut off by the plane $\vec{r}.\left(2\,\hat{\imath}+\hat{\jmath}-\hat{k}\right)-5=0$ on the three axes. **Ans:** $\frac{5}{2}$

21. If $\vec{a}$ and $\vec{b}$ are unit vectors, then what is angle between $\vec{a}$ and $\vec{b}$ for $\vec{a}-\sqrt{2}\,\vec{b}$ to be a unit vector? **Ans:** $\frac{\pi}{4}$

22. Find the general solution of the differential equation: $\frac{dy}{dx}=\sqrt{4-y^2}$, $-2<y<2$ **Ans:** $\boldsymbol{y=2sin(x+C)}$

23. Using integration, find the area bounded by the lines $2y=-x+8$, $x=2$ and $x=4$ **Ans: 5 sq units**

24. Evaluate: $\int_{-2}^{2}\frac{x^2}{1+5^x}\mathrm{d}x$ **Ans:** $\frac{16}{3}$

(**OR**)

If $\int_0^a \frac{1}{4+x^2}\,dx=\frac{\pi}{8}$, find the value of a. **Ans:2**

25. Find the slope of the normal to the curve $y=x^2-\frac{1}{x^2}$ at $(-1,0)$ **Ans:4**

26. Show that the given function is not differentiable at $x=2$

$$f(x)=\begin{cases} x-1, & \text{if } x<2 \\ 2x-3\ , & \text{if } x\geq 2 \end{cases}$$

27. For what values of k, the system of linear equations

$\boldsymbol{x}+\mathrm{y}+\mathrm{z}=2,\ 2\boldsymbol{x}+\mathrm{y}-\mathrm{z}=3\ ,\ \ 3\boldsymbol{x}+\boldsymbol{2}\mathrm{y}+\ \mathrm{k}\,\mathrm{z}=4$ has a unique solution? **Ans:** $\boldsymbol{k=R-\{0\}}$

(**OR**)

If $A=\begin{bmatrix}\cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha\end{bmatrix}$, then for what value of α, is A an identity matrix ? **Ans:0**

28. Consider a function $f: R_+ \to [4,\infty)$ is given by $f\,(x)=x^2+4$. Show that f is one-one and onto.

SECTION- IV [3 marks each]

29. Let f: X→Y be a function.Define a relation R in X given by R=$\{(a,\mathrm{b}):\mathrm{f}(a)=\mathrm{f(b)}\}$. Examine whether R is an equivalence relation or not..

30. If $y=e^{a\cos^{-1}x}$, $-1\leq x\leq 1$ then show that: $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}-a^2y=0$

(**OR**)

If $x^{16}\,.\ y^9=(x^2+y)^{17}$, prove that $\frac{dy}{dx}=\frac{2y}{x}$

31. If $y=\log\left\{x+\sqrt{x^2+a^2}\right\}$ prove that: $(x^2+a^2)\frac{d^2y}{dx^2}+x\frac{dy}{dx}=0$

32. Evaluate: $\int\frac{1}{1-\tan x}dx$ **Ans:** $\boldsymbol{\frac{x}{2}-\frac{1}{2}log|cos\,x-sin\,x\,|+c}$

(**OR**)

Evaluate: $\int_0^{\frac{\pi}{4}}\log(1+\tan x)\,dx$ **Ans:** $\boldsymbol{\frac{\pi}{8}log\,2}$

33. Show that $y=\log(1+x)-\frac{2\,x}{2+x}$, $x>-1$ is an increasing function of *x* throughout its domain.

34. Find the area of bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. **Ans: πab sq. units**

35. Solve the differential equation :$x\frac{dy}{dx} = y - x\tan\left(\frac{y}{x}\right)$ **Ans:** $\boldsymbol{x sin\frac{y}{x} = c}$

SECTION- V [5 marks each]

36. Find the coordinates of the image of the point $(1, 3, 4)$ in the given plane $2x - y + z + 3 = 0$
Ans: $(\boldsymbol{-3, 5, 2})$

(OR)

Show that the lines: $\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$ and $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$are coplanar. Find this plane also
Ans: $\boldsymbol{x - 2y + z = 0}$

37. Solve the following linear programming problem $(\mathrm{L.P.P.})$graphically:
Minimize Z= $3x + 9\,y$ Subject to the constraints: $x + 3y \leq 60$, $x + y \geq 10$, $x \leq y$, $x \geq 0$, $y \geq 0$ **Ans: 60 at**$(\boldsymbol{5},\ \boldsymbol{5})$

(OR)

One kind of cake requires 200 g of floor and 25 g of fat, and another kind of cake requires 100 g of floor and 50 g of fat. Find the maximum number of cakes which can be made from 5 kg of floor and 1 kg of fat assuming that there is no shortage of the ingredients used in making the cakes. **Ans: 30,10**

38. If A= $\begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 2 \\ -3 & 1 & -1 \end{bmatrix}$, find $\mathrm{A}^{-1}$and hence solve the given equations:

$2x + y - 3z = 13$, $3x + 2y + z = 4$, $x + 2y - z = 8$ **Ans:** $\boldsymbol{x = 1, y = 2, z = -3}$

(OR)

The cost of 4 kg onion, 3 kg wheat and 2 kg rice is Rs. 60. The cost of 2 kg onion, 4 kg wheat and 6 kg rice is Rs. 90. The cost of 6 kg onion, 2 kg wheat and 3 kg rice is Rs. 70. Find the total cost of 2 kg onion, 3 kg wheat and 1 kg rice. (Using matrix method).
Ans: 1kg onion @ Rs 5, 1kg wheat @ Rs 8, 1kg rice @ Rs 8 , total cost of 2 kg onion, 3 kg wheat and 1 kg rice is Rs.42

Kendriya Vidyalaya Sangathan Raipur -Region

Time: 3 Hours **Sample Question Paper- V (2020 – 21)** M. M: 80

Class-XII

Subject: - Mathematics

General Instructions:--

4. This question paper contains two **Parts A** and **B**. Each part is compulsory. Part **A** carries **24** marks and Part **B** carries **56** marks.
5. **Part-A** has Objective Type Questions and **Part-B** has Descriptive Type Questions
6. Both Part A and Part B have choice.

Part-A

4. It consists of two sections- **I** and **II**
5. Section **I** comprises of **16** very short answer type questions.
6. Section **II** contains **2** case studies. Each case study comprises of 5 case –based MCQs. An examinee is to attempt **any 4 out of 5 MCQs.**

Part-B

6. It consists of three sections-**III,IV** and **V.**
7. Section **III** comprises of 10 questions of **2 marks** each.
8. Section **IV** comprises of 7 questions of **3 marks** each.
9. Section **V** comprises of 3 questions of **5 marks** each.
10. Internal choice is provided in **3** questions of section- III, **2** questions of section- IV and **3** questions of section- V. You have to attempt only one of the alternatives in all such questions.

$[\textbf{Part} - \textbf{A}]$

SECTION- I[1 mark each]

1. Let A = {1, 2} and B = {1, 3} and R be the relationfrom set A to set B defined as R = {(1, 1), (1, 3),
(2, 1), (2, 3)}. Is R a universal relation? Give reasons..**Ans:**$\{\textbf{yes}, \textbf{as } \mathbf{R} = \mathbf{A} \times \mathbf{B}\}$

2. If set A has 3 elements and set B has 4 elements thenfind the number of injective mapping from A to B**Ans:** $\mathbf{4_{P_3} = 24}$

.(**OR**)

Let *f:* R→R be defined by, $f(x) = x^2 + 1$ thenfind the pre-mapping of 17 and -3.**Ans:** $\pm\mathbf{4}, \emptyset$

3. Consider the set A= $\{1, 2, 3\}$ and R be thesmallest equivalence relation on A. Write thesmallest equivalence relation.**Ans:**$\{(\mathbf{1},\mathbf{1}),(\mathbf{2},\mathbf{2}),(\mathbf{3},\mathbf{3})\}$

4. If $x \in$ Nand $\begin{vmatrix} x+3 & -2 \\ -3x & 2x \end{vmatrix} = 8$, then find the value of x.**Ans: 2**

5. Find the matrix X for which $\begin{bmatrix} 1 & -4 \\ 3 & -2 \end{bmatrix} X = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$. **Ans:**$\boldsymbol{X} = \begin{bmatrix} 6 & 2 \\ \frac{11}{2} & 2 \end{bmatrix}$

6. Matrix A $= \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$ is given to be symmetric, find the values of a and b**Ans:** $\boldsymbol{a} = \frac{-2}{3}, \boldsymbol{b} = \frac{3}{2}$

(**OR**)

If A is a square matrix of order 3 and $|4\,\mathrm{A}| = \mathrm{K}|\mathrm{A}|$, then write the value of K **Ans: 64**

7. Evaluate : $\int \tan x \sqrt{1+\tan^2 x}\, dx$. **Ans:** $\frac{2}{3}(\tan x)^{\frac{3}{2}} + C$

(**OR**)

Evaluate : $\int \sin 3x . \sin 2x\, dx$. **Ans:** $\frac{\sin x}{2} - \frac{\sin 5x}{10} + C$

8. Find the area bounded by the curve $y = \sqrt{3x+4}$,x-axis, $x = 0$ and $x = 4$ **Ans:** $\frac{112}{9}$ **sq. units**

9. If m and n are the order and degree, respectively of the differential equation $y\left(\frac{dy}{dx}\right)^3 + x^3\left(\frac{d^2y}{dx^2}\right)^2 - xy = \sin x$, then write the value of m + n **Ans:4**

(OR)

Write the integrating factor of differential equation: $(\tan^{-1}y - x)dy = (1+y^2)dx$ **Ans:** $e^{\tan^{-1}y}$

10. Show that the vector $\hat{\imath} + \hat{\jmath} + 2\hat{k}$ are equally inclined to the axes OX,OY and OZ

(**OR**)

Find angle θ between the vectors $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\imath} - \hat{\jmath} - \hat{k}$ **Ans:** $\cos^{-1}\left(\frac{-1}{3}\right)$

11. For what values of μ are the vectors $\vec{a} = 2\,\hat{\imath} + \mu\hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\imath} - 2\,\hat{\jmath} + 3\,\hat{k}$ perpendicular to each other.? **Ans:** $\frac{5}{2}$

12. Find λ, when the projection of $\vec{a} = \lambda\hat{\imath} + \hat{\jmath} + 4\hat{k}$ on $\vec{b} = 2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}$ is 10 units. **Ans:** $\lambda = 26$

13. Find the intercepts cut off by the plane $3x - 2y - 5z = 5$ on the three axes. **Ans:** $\frac{5}{3}, -\frac{5}{2}. -1$

14. Find the Cartesian equation of the line which passes through the point $(-2, 4, -5)$ and is parallel to the line $\frac{x+3}{3} = \frac{4-y}{5} = \frac{z+8}{6}$ **Ans:** $\frac{x+2}{3} = \frac{y-4}{-5} = \frac{z+5}{6}$

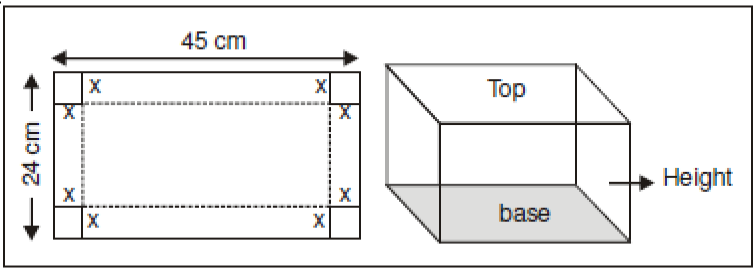
15. If A and B are two independent events with $P(A) = 0.3$ and $P(B) = 0.4$, then find the value of (i) $P(A \cap B)$ (ii) $P(A \cup B)$ **Ans: (i) 0.12 (ii) 0.58**

16. A coin is tossed twice. Find the probability distribution of number of heads. **Ans:**

X	**0**	**1**	**2**
P(X)	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$

SECTION- II For Q.17 and Q18 attempt any four (MCQs) [1 mark each]

17. Mr. Saurabh wants to perform an activity using acoloured paper sheet of size 45 cm × 24 cm. Hecuts off squares of side x cm from each corner andfold the flaps upward to form a open box as shown in figure.



On the basis of above informations answer thefollowings

(i) The volume of the open box in terms of x is:

(a) $(45-x).(24-x)x$ **(b)**$(45-2x).(24-2x)x^2$ **(c)**$(45-2x)(24-2x)2x$

(d)$(45-2x)(24-2x)x$ **Ans : (d)**

(ii) If $\frac{dV}{dx}=0$ then possible value of x is (V is volume of box).

(a) 18**(b)** 14 **(c)** 5 **(d)** 10**Ans : (c)**

(iii) Maximum volume of box is:

(a)$3500\ cu\ cm$ **(b)** $2750\ cu\ cm$ **(c)** $1450\ cu\ cm$ **(d)**$2450\ cu\ cm$ **Ans : (d)**

(iv) Area of rectangular sheet is equal to:

(a) $1400\ sq\ cm$ **(b)** $1080\ sq\ cm$ **(c)** $1380\ sq\ cm$ **(d)**$576\ sq\ cm$ **Ans : (b)**

(v) For volume of box to be maximum$\frac{d^2V}{dx^2}$ will be

(a)$-ve$**(b)** $+ve$**(c)** **0(d)** none of these **Ans : (a)**

18. Three persons A, B and C apply for a job in aprivate school for the post of principal. The chances of their selection are in the ratio 2 : 3 : 4respectively.Management committee given the agenda toimprove the sports education, it is estimated thatthe change may occur with probability 0.8, 0.5and 0.3 respectively.

On the bases of above situation answer thefollowings:

(i) The probability of A not selected is

(a) $\frac{2}{9}$**(b)** $\frac{7}{9}$ **(c)** $\frac{3}{9}$ **(d)** $\frac{4}{9}$**Ans : (b)**

(ii) Probability of 'C' that change not take place is

(a)$\frac{2}{10}$**(b)** $\frac{5}{10}$ **(c)** $\frac{7}{10}$ **(d)** $\frac{3}{10}$ **Ans : (c)**

(iii) The probability of selection of C is

(a) $\frac{4}{9}$**(b)** $\frac{6}{9}$ **(c)** $\frac{3}{9}$ **(d)** $\frac{1}{9}$ **Ans : (a)**

(iv) Probability of 'A' that change occur is

(a)$\frac{8}{10}$**(b)** $\frac{5}{10}$ **(c)** $\frac{3}{10}$ **(d)** $\frac{2}{10}$ **Ans : (a)**

(v) Probability of 'B' that change not take place is

(a)$\frac{3}{10}$**(b)** $\frac{5}{10}$ **(c)** $\frac{3}{10}$ **(d)** $\frac{1}{10}$ **Ans : (b)**

[Part – B]

SECTION- III [2 marks each]

19. Let A $= \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$, then show that $A^2 - 4A + 7I = 0$.

20. Prove that: $\sin[\cot^{-1}\{\cos(\tan^{-1}x)\}] = \sqrt{\frac{x^2+1}{x^2+2}}$

(OR)

If$4\sin^{-1}x + \cos^{-1}x = \pi$, then find the value$x$**Ans :**$\frac{1}{2}$

21. If $f\ (x) = \begin{cases} \frac{\sin 2x}{x}, & x \neq 0 \\ \text{k}, & x = 0 \end{cases}$ is continuous at $x = 0$ then find the value of k.**Ans**: $\mathbf{2}$

22. Find the intervals in which the function f $(x) = x^3 - 3x^2 - 72x + 18$ is

(a) strictly increasing (b) strictly decreasing. **Ans**: $(\mathbf{a})(-\infty, \mathbf{-4}) \cup (\mathbf{6}, \infty)(\mathbf{b})(\mathbf{-4}, \mathbf{6})$

23. Out of 8 outstanding students of a school, in which there are 3 boys and 5girls , a team of 4 students is to be selected for a quiz completion. Find the probability that 2 boys and 2 girls are selected.**Ans** $\mathbf{\frac{3}{7}}$

24. Evaluate:$\int \frac{e^x(1+x)}{\cos^2(e^x x)} dx$ **Ans:**$\boldsymbol{tan}(e^x x) + \boldsymbol{c}$

(**OR**)

Evaluate: $\int_1^4 |x - 2|\, dx$ **Ans:** $\mathbf{\frac{5}{2}}$

25. Evaluate: $\int \frac{1}{\sqrt{5-2x-x^2}} dx$ **Ans:**$\boldsymbol{sin}^{-1}\left(\frac{\boldsymbol{x}+\mathbf{1}}{\sqrt{\mathbf{6}}}\right) + \boldsymbol{c}$

26. Solve the differential equation :$\frac{dy}{dx} + y = \cos x - \sin x$.**Ans:**$\boldsymbol{y} = \boldsymbol{cos\ x} + \boldsymbol{Ce}^{-\boldsymbol{x}}$

27. If P(2,3,4) is the foot of perpendicular from origin to a plane, then write the vector equation of this plane. **Ans** :$\vec{\boldsymbol{r}}.\left(\mathbf{2}\hat{\boldsymbol{\imath}} + \mathbf{3}\hat{\boldsymbol{\jmath}} + \mathbf{4}\hat{\boldsymbol{k}}\right) = \mathbf{29}$

(**OR**)

Show that the lines$\vec{r} = (-3\,\hat{\imath} + \hat{\jmath} + 5\,\hat{k}\,) + \lambda\left(-3\hat{\imath} + \hat{\jmath} + 5\hat{k}\right)$and

$\vec{r} = \left(-\,\hat{\imath} + 2\hat{\jmath} + 5\hat{k}\right) + \mu\left(-\,\hat{\imath} + 2\hat{\jmath} + 5\hat{k}\right)$are coplanar.

28. If $\hat{a}, \hat{b}$ and $\hat{c}$ are mutually perpendicular vectors , then find the value of$\left|2\hat{a} + \hat{b} + \hat{c}\right|$**Ans:**$\sqrt{\mathbf{6}}$

SECTION- IV [3marks each]

29. Evaluate: $\int_0^{\frac{\pi}{2}} \frac{1}{1+\sqrt{\cot x}} dx$ **Ans:** $\boldsymbol{\frac{\pi}{4}}$

(**OR**)

Evaluate:$\int \frac{1}{\cos(x-a)\cos(x-b)} dx$ **Ans:** $\frac{\mathbf{1}}{\boldsymbol{sin}\ (\boldsymbol{a}-\boldsymbol{b})}\ \boldsymbol{log}\ \left|\frac{\boldsymbol{cos}\ (\boldsymbol{x}-\boldsymbol{a})}{\boldsymbol{cos}\ (\boldsymbol{x}-\boldsymbol{b})}\right| + \boldsymbol{c}$

30. If$\sqrt{1 - x^2} + \sqrt{1 - y^2} = a(x - y)$, show that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$

(**OR**)

If $x = a\left(\cos\theta + \log\tan\frac{\theta}{2}\right)$ and $y = a\ \sin\theta$, find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$ **Ans:**$\frac{\mathbf{2}\sqrt{\mathbf{2}}}{\boldsymbol{a}}$

31. Consider f : R$-\left\{-\frac{4}{3}\right\} \rightarrow$ R$-\left\{\frac{4}{3}\right\}$ given by $f(x) = \frac{4x+3}{3x+4}$ .Show that f is bijective.

32. If y =P e^{ax} +Qe^{bx} ,show that $\frac{d^2y}{dx^2} - (a+ \text{b})\frac{dy}{dx} + a\ \text{b}\ y = 0$

(**OR**)

Let $f\,(\boldsymbol{x})$=$\begin{cases} \frac{\mathbf{1}-\mathbf{cos}\,\mathbf{4}\boldsymbol{x}}{\boldsymbol{x}^{\mathbf{2}}}, & \mathbf{if}\boldsymbol{x} < 0 \\ \boldsymbol{a}\ , & \mathbf{if}\boldsymbol{x} = \mathbf{0} \\ \frac{\sqrt{\boldsymbol{x}}}{\sqrt{\mathbf{16}+\sqrt{\boldsymbol{x}}}-\mathbf{4}}, & \mathbf{if}\boldsymbol{x} > 0 \end{cases}$ **Ans:**$\boldsymbol{a} = \mathbf{8}$

Determine the value of 'a' so that f $(\boldsymbol{x})$ is continuous function at $\boldsymbol{x} = \mathbf{0}$

33. Find the particular solution of the differential equation: $\frac{\mathrm{dy}}{\mathrm{d}x} = \frac{xy}{x^2+y^2}$ given that $y = 1$when $x = 0$ **Ans:**$\boldsymbol{log y = -\frac{x^2}{2y^2}}$

34. Find the equation of the tangent to the curve $y = \sqrt{3x-2}$ which is parallel to the line $4x - 2y + 5 = 0$ **Ans: :** $\mathbf{48x - 24y - 23 = 0}$

35. Find the area of the region bounded by the parabola $y = x^2$ and $y = |x|$**Ans:** $\frac{\mathbf{1}}{\mathbf{3}}$**sq.units**

(**OR**)

Find the area of bounded by the ellipse$\frac{x^2}{36} + \frac{y^2}{25} = 1$. **Ans: 30π** $\mathbf{sq. units}$

SECTION- V [5marks each]

36. Solve the following linear programming problem(L. P. P.)graphically:

Maximize Z= $\frac{15}{2}x + 5y$ Subject to the constraints: $2x + y \leq 60$, $2x + 3y \leq 120$, $x \leq 20$, $x \geq 0$, $y \geq 0$**Ans: 262.50 at**$(\mathbf{15}, \mathbf{30})$

(OR)

Solve the following linear programming problem(L. P. P.)graphically:

Maximize: Z= $5x + 2y$ Subject to the constraints: $x - 2y \leq 2$, $3x + 2y \leq 12$, $-3x + 2y \leq 3$, $x \geq 0$, $y \geq 0$**Ans: 19 at**$\left(\frac{\mathbf{7}}{\mathbf{2}}, \frac{\mathbf{3}}{\mathbf{4}}\right)$

37. Using matrix, solve the following system of equations :

$x + 2y - 3z = -4$, $2x + 3y + 2z = 2$, $3x - 3y - 4z = 11$**Ans:**$\mathbf{x = 3, y = -2, z = 1}$

(OR)

Find the inverse of matrix A = $\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$and hence show that $\mathrm{A}^{-1}.\mathrm{A} = \mathrm{I}$

Ans: $\mathbf{A^{-1}} = \begin{bmatrix} \mathbf{2} & \mathbf{0} & \mathbf{-1} \\ \mathbf{5} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{3} \end{bmatrix}$

38. Find the distance of the point $(2, 12, 5)$,from the point of intersection of the line $\vec{r} = (2\hat{\imath} - 4\hat{\jmath} + 2\hat{k})+\beta(3\hat{\imath} + 4\hat{\jmath} + 2\hat{k})$ and the plane$\vec{r} . (\hat{\imath} - 2\hat{\jmath} + \hat{k}) = 0$.**Ans: 13**

(OR)

Find the coordinates of the foot of the perpendicular and perpendicular distance of the point (1,3,4) from the plane $2x - y + z + 3 = 0$.Find also, the image of the point in the plane. **Ans:**$(-\mathbf{1}, \mathbf{4}, \mathbf{3})$,$\sqrt{\mathbf{6}}$,$(\mathbf{-3}, \mathbf{5}, \mathbf{2})$

तत् त्वं पूषन् अपावृणु
केन्द्रीय विद्यालय संगठन

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